Sentiments, Financial Markets, and Macroeconomic Fluctuations*

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Abstract

This paper studies how financial information frictions can generate sentiment-driven fluctuations in asset prices and self-fulfilling business cycles. In our model economy, exuberant financial market sentiments of high output and high demand for capital increase the price of capital, which signals strong fundamentals of the economy to the real side and consequently leads to an actual boom in real output and employment. The model further derives implications for asymmetric non-linear asset prices and for economic contagion and co-movement across countries. In the extension to the dynamic OLG setting, our model demonstrates that sentiment shocks can generate persistent output, employment and business cycle fluctuations, and offers some new implications for asset prices over business cycles.

JEL classification: G01, G20, E2, E44

Keywords: Financial sector; Real economy; Feedback; Sentiment-driven fluctuations; Self-fulfilling business cycles; Beauty contests; Animal spirits

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1 Introduction

The financial sector plays a central role in the modern economy, as is evident from the wide and deep macroeconomic impact of the recent global financial crisis in 2007-2009. There are at least two channels through which the financial sector can influence the aggregate real economy (see, e.g., Levine (2005)): 1) the financing of capital; 2) the production of information about investment opportunities. An exploding financial accelerator literature has shown, both theoretically and empirically, that the financial sector can influence business cycles through the *financing* channel.[1] In this paper we explore the feedback effect from the financial market to the real economy due to the *informational* role of financial prices. Unlike the conventional view that prices can help to efficiently allocate economic resources in a free market by signaling relevant information to economic actors (Hayek (1945) and Grossman and Stiglitz (1980)), we argue that the informational role of financial markets in allocating resources can be impaired by investors' sentiments or sunspots. The sentiment-driven asset prices in turn may influence real activities and shape macroeconomic fluctuations.

We are motivated by a large empirical finance literature that has documented that investor sentiment in financial markets can affect asset prices (see, e.g., the surveys by Hirshleifer (2001) and Baker and Wurgler (2007)). The aggregate (macro)-level asset prices are in particular sensitive to investor sentiment, which in turn impacts corporate financing and investment (Lamont and Stein (2006)). The recent empirical work of Angeletos, Collard and Dellas (2014) also finds that business cycle fluctuations can be attributed to sentiments. Levchenko and Pandalai-Nayar (2015) identify the sentiment shock as being more important than other factors in explaining business cycle comovement between the US and Canada.

We formalize our idea in a simple baseline three-period *rational expectations model* consisting of a continuum of investors and workers. The investors live from period 0 to period 1. They are the initial capital owners. The workers live from period 1 to period 2. The only fundamental uncertainty in the economy is the *aggregate* total factor productivity (TFP) shock in the last period (period 2). We assume, in the baseline case, that only the investors have information about the TFP shock. The TFP shock in period 2 directly affects the workers' return on capital holdings in period 2 (which are their labor income savings from period 1) and hence their incentive to supply labor in period 1. As capital and labor are complements in production, the workers' labor supply in period 1 in turn affects the investors' return on capital held from period 0 to period 1. In such an economic environment, the investors in period 0 will need to forecast the level of aggregate economic activity, that is, employment and output in period 1. On the other side, forming expectations about the

[1] See, e.g., the seminal work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) and a recent excellent survey by Brunnermeier et al. (2013).

behavior of the investors, the workers can obtain information from the price of capital in period 0 about the return on their capital savings for period 2. This two-way interaction between the financial market and the real economy is at the heart of our mechanism of sentiments.

Suppose that somehow exuberant sentiments lead the investors to believe there will be a boom in output in period 1. Then they conjecture that the demand for capital and therefore return on capital will be high. Competition in the financial market will push up the capital price in period 0. However, the workers cannot tell whether the high capital price is due to the investors' sentiments or their signal of a high TFP in period 2. After solving a signal extraction problem, they will attribute the high price partially to a high TFP in period 2. Their actual labor supply will indeed increase, resulting in an actual boom in output in period 1. So the investors' initial belief will be confirmed. We show that there exist sentiment-driven equilibria, in which the capital price reflects both sentiment and TFP shocks. Under these rational expectations equilibria, Bayesian optimal signal extraction will result in an actual labor supply of workers that is always equal to investors' conjectured labor supply.

The sentiment-driven fluctuation studied in our baseline model links the Keynesian notions of "beauty contests" and "animal spirits". What matters to an individual investor is not his own assessment of the fundamentals, but his conjecture about the actions of other investors, as in a standard beauty contest game. Under the feedback effect, the asset price can influence real decisions and generate complementarities between the actions of investors. Thus, the sentiment shocks in financial markets endogenously drive the fundamentals and generate aggregate output fluctuations.

In our framework of the macroeconomy with feedback effects, we also derive implications for asymmetric non-linear asset prices and for economic contagion and co-movement across countries. First, we show that our sentiment-driven equilibria can be non-linear: When the fundamental value is high, the asset price only reflects fundamentals; when the fundamental value is low, the asset price is driven by both fundamentals and sentiments. Essentially, if investors perceive that the real side of the economy is affected by sentiments only for low fundamentals, their beliefs can become self-fulfilling under the two-way feedback. In such non-linear equilibria, the price informativeness is asymmetric in fundamentals and the asset price exhibits a large discontinuity such that asset price collapses occur sometimes with a small change in economic fundamentals. This can help explain some asset price puzzles, as for example documented by Culter et al. (1989). Second, empirical evidence suggests that asset price contagion cannot be explained by fundamentals.[2] A prominent feature of the recent Great Recession is that it was global, even affecting many emerging countries with heavy capital controls. Perri and Quadrini (2013) document that all major industrialized countries experienced extraordinarily large and unprecedentedly synchronized contractions

[2] See, e.g., the findings of Karolyi and Stultz (1996) and King and Wadhwani (2000).

in output and asset prices during the Great Recession. Our model is able to characterize such synchronization. Due to the informational feedback between the financial market and the real economy, investors' perception of synchronization across countries can lead to actual synchronization.

Finally, we extend our baseline model to a dynamic setting of an overlapping generations (OLG) model. In the dynamic setting, the current savings of workers become the capital stock in the subsequent period. The capital stock therefore is dynamically linked across periods through savings. In the sentiment-driven equilibria, capital accumulation, as well as output and employment, is driven not only by the private future productivity signals received by investors, but also by their sentiments. Hence, i.i.d. sentiment shocks can generate persistent fluctuations in output and unemployment. As persistence is a defining feature of all business cycles, this extension illustrates that sentiments also hold the promise of explaining the persistence in real data. While building a full DSGE model and confronting it with data is beyond the scope of this paper, the mechanisms developed herein can lay the ground for such work.

The OLG model also generates a number of predictions about asset prices over the business cycle. First, we show, with a closed-form solution, that the risk premium is increasing in sentiment volatility. When the sentiment volatility increases, the asset price contains noisier information about future fundamental shocks and hence the investors demand a higher premium on the risky investment. This implies that an economy with higher investor sentiment volatility in financial markets will have a higher risk premium in asset returns. Emerging markets, for example, are more likely to experience higher sentiment volatility because of lower transparency in information disclosure. This may partially explain why these countries typically have larger risk premia than the developed economies (see Salomons and Grootveld (2003) for the empirical evidence). The same argument also implies that as information transparency improves in the same country over time, the risk premium will decline. This is consistent with the evidence showing that the equity premium has declined (Lettau et al. (2008), Fama and French (2002)) after the enactment of new disclosure requirements in 1980 (Fox et al. (2003)). Second, our model shows that time-varying sentiment volatility yields a time-varying risk premium. Several empirical studies have documented that investors' sentiment may be affected by the change of seasons.[3] The seasonal change in the sentiment volatility thus can generate seasonal self-fulfilling equilibrium fluctuations in the risk premium. Our model hence provides a rational framework to explain the financial market seasonality, which is in general regarded as a market anomaly in the context of the efficient market hypothesis.[4] Interestingly, the seasonality in asset returns is primarily a small-firm phenomenon,

[3] Saunders (1993) finds that the number of hours of sunshine affects people's mood and hence market returns. Hirshleifer and Shumway (2003) provide some international evidence on the sunshine effect. Kamstra, Kramer and Levi (2003) provide further compelling evidence of a link between the seasonal depression due to the seasonal affective disorder (SAD, also known as winter blues or winter depression) and seasonal variations in stock returns.

[4] See De Bondt and Thaler (1987) for an excellent survey of the empirical evidence.

consistent with our theoretical prediction. Baker and Wurgler (2006) document that smaller firms are more likely to be affected by investor sentiments.

Related literature. Our paper relates to several strands of literature. First, our paper adds to the growing recent literature that studies the feedback effect from financial markets to the real side of the economy due to informational frictions. A number of contributions to this literature use a partial equilibrium model to study one firm or a de-facto-one-firm aggregate economy. For example, a firm manager obtains information about the return on his own firm's investment (typically exogenously given) from financial markets. Bond, Edmans and Goldstein (2012) provide an extensive survey of this literature.[5] Luo (2005), Chen et al. (2007), Bakke and Whited (2010), Foucault and Fresard (2014), among others, provide empirical evidence for the feedback effect. Our work brings this growing micro literature on informational feedback effects to a macroeconomic model. In our model with a general equilibrium framework, agents form expectations and undertake investments based on information from financial markets about the aggregate state of the economy. A key feature of our model therefore is that the noisy information and prices are correlated through general sentiments about the aggregate economy, and can generate non-fundamental rational expectations equilibria.[6] We believe that our study of the feedback effect operating through the macroeconomy is important. In fact, when firms decide how much to produce, the market demand for their goods would be heavily influenced by the level of aggregate demand and the state of the economy. On the other hand, the financial price indexes, which reflect forward-looking views of most sophisticated investors, are widely seen as a barometer of the aggregate economy.

Our paper is closely related to Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren and Yuan (2013). These papers also study the interaction between the real sector and the financial market. In Angeletos, Lorenzoni and Pavan (2010), information spillover flow from the real sector to the financial sector, which can generate a strategic complementarity in investment, amplify non-fundamental shocks, and create multiple market equilibria under certain conditions. The non-fundamental shocks in their model come from the correlated errors in information about the fundamentals. In contrast, the non-fundamental shocks in our model come from investors' sentiments. We establish the existence of a continuum of sentiment-driven equilibria and also study nonlinear asymmetric sentiment-driven equilibria. In fact, a long tradition in macroeconomics has resorted to models that feature multiple equilibria to explain "non-fundamental" fluctuations in

[5] For the theoretical work, see, e.g., Fishman and Hagerty (1992), Leland (1992), Dow and Gorton (1997), Subrahmanyam and Titman (1999, 2013), Hirshleifer et al. (2006), Foucault and Gehrig (2008), Goldstein and Guembel (2008), Ozdenoren and Yuan (2008), Bond et al. (2010), Kurlat and Veldkamp (2012), Sockin and Xiong (2012), Goldstein and Yang (2013), and Huang and Zeng (2014).

[6] The multiplicity of equilibria in our model may be understood in terms of the correlated equilibria induced by market sentiments. See Aumann (1987), Maskin and Tirole (1987), Aumann, Peck and Shell (1988), Peck and Shell (1991), Bergemann and Morris (2011), and Benhabib, Wang and Wen (2015). Correlated signals can coordinate actions of firms and of workers to produce additional rational expectations equilibria.

terms of "animal spirits". Equally importantly, we apply our mechanism to the international synchronization of business cycles across countries as well as dynamic OLG economies, which provides a novel channel to explain how financial markets can affect business cycle fluctuations.[7] Goldstein, Ozdenoren and Yuan (2013) study information spillovers from the financial market to the real sector,[8] as in our paper. "Trading frenzies" can arise in their model as the capital provider in the real sector optimally extracts information about investment returns from the financial price driven by the speculators' correlated signals. The key mechanism of their model is that the informational feedback between the financial market and the real sector can generate complementarities in trading of speculators, namely, speculators all wish to trade like others. The authors introduce noise traders and focus on parameters that give a unique equilibrium, different from ours.[9]

Second, our work is related to a set of papers that emphasize informational frictions in explaining asset price puzzles. Yuan (2005) and Barlevy and Veronesi (2003) explain the asymmetric and nonlinear response in asset prices in a framework of informational frictions in an exchange economy. We show that our sentiment-driven equilibria can also generate such asymmetric responses in asset prices in a production economy. Gaballo (2013) shows that the introduction of an arbitrarily small degree of price dispersion can generate large departures from the perfect-information benchmark. Like ours, his model is fully microfounded. His mechanism crucially depends on dispersed information. Equilibrium multiplicity in our model exists even without dispersed information and instead stems from the two-way feedback between the financial market and the real economy. Benhabib and Wang (2013) construct a sequential trading model without noise traders, in which short-term traders condition their trades both on private signals from fundamentals and on sunspots. Investors purchase the assets in centralized markets using market prices to form Bayesian expectations about final period returns. Benhabib and Wang (2013) show the existence of a continuum of non-fundamental sunspot equilibria. Our paper differs from theirs in that the connection between sentiments and the real economy is absent in their paper and they do not address feedback effects from the financial market to the real economy arising from informational frictions.

Third, our paper is related to some other recent work on self-fulfilling business cycles, which has generated renewed interest after the recent financial crisis.[10] Perri and Quadrini (2013) use self-

[7] Allen, Morris and Shin (2006) also study market structures with sequential trading by differentially informed short-horizon traders who receive noisy public signals. They show that the public signals can indeed be over-weighted by short-term traders interested in predicting average expectations relative to the private information of final payoffs, giving rise to a Keynesian beauty contest in market prices. See also Morris and Shin (2002).

[8] See also Goldstein, Ozdenoren and Yuan (2011) and Goldstein and Yang (2013).

[9] See also Albagli, Hellwig and Tsyvinski (2013) where informed and uninformed traders face limits on their asset positions. Demand fluctuations from realizations of fundamentals, or from noise traders, alter the identity of the marginal investor. This can drive a wedge between prices and expected returns from the perspective of an outsider, and generate excess price volatility relative to fundamentals. Albagli, Hellwig and Tsyvinski (2014) endogenize security cash flows as the outcome of firm decisions.

[10] Using a different approach with bilateral trades and Bayesian updating of information, Angeletos and La'O (2012) also derive sentiment-driven business cycles without using sunspots or multiple equilibria.

fulfilling expectations to explain a global recession in a two-country model with financial integration. Similarly, Bacchetta and Wincoop (2013) construct a two-period two-country model with both a financial linkage and a trade linkage. The self-fulfilling beliefs in their model rely on a real complementarity between future and current output. These papers do not, however, study the two-way interaction between the financial market and the real economy emphasized in our paper. In a closed-economy setting, Benhabib, Wang and Wen (2015, 2013) study self-fulfilling business cycles in a modified Dixit-Stiglitiz monopolistic competition model. In their model, firms receive quantity signals on their idiosyncratic demand and aggregate output. Firms, in optimally choosing their output, observe their signal and partially attribute it to aggregate demand, which then becomes self-fulfilling. Financial markets play no role in their model. In contrast, our paper emphasizes the two-way feedback between the financial market and the real economy and shows that the financial markets can be a source of endogenous signals generating self-fulfilling fluctuations.

The paper is organized as follows. Section 2 lays out the baseline model and Section 3 presents the equilibria. Section 4 generalizes the baseline model by allowing more general information structures. Section 5 studies further implications of the model on non-linear asset prices and economic contagion and co-movement. Section 6 extends the model to a dynamic economy setting. Section 7 concludes.

2 The Baseline Model

We start with a three-period baseline model with a financial sector and a real sector. The financial sector consists of a continuum of investors with unit mass. The real sector has a representative competitive firm and a continuum of workers with unit mass. The investors live from period 0 to period 1 but only consume in period 1. Each investor is endowed with $K_0 = 1$ unit of capital in period 0. Investors trade their capital in the financial market with price P_0 in period 0. Each unit of capital will allow its owner to receive R_1 units of dividend (final goods) in period 1, where R_1 will be endogenized. The workers live from period 1 to period 2 but only consume in period 2. The workers supply labor in periods 1 and 2 to the competitive firm. The workers use their wage income in period 1 to purchase final goods to save, thereby becoming the owners of capital in period 2. The competitive firm combines capital and labor to produce final goods that can be used both for consumption and as new capital according to the production function

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{1}$$

where A_t is productivity (TFP), K_t is the firm's capital input and N_t is the firm's labor input in period $t = 1, 2$. Capital fully depreciates after production in each period.

The Firm The firm solves a trivial problem. Let W_t and R_t be the real wage and the rental price (dividend) of capital, respectively. The profit maximization yields

$$\begin{aligned} W_t &= (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha}, \\ R_t &= \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha}. \end{aligned}$$

Financial Market and Information Structure The financial market opens in period 0 and the investors trade their capital among themselves, based on their private information. Trading capital is equivalent to trading shares of firms in the financial market. That is, equivalently, the representative firm has $K_0 = 1$ unit of shares, each of the continuum of investors holds $K_0 = 1$ unit of shares in period 0, and each share receives R_1 units of dividend (final goods) in period 1; hence, the capital price can be interpreted as the equity (share) price of the representative firm. The only fundamental uncertainty in the economy is A_2. Specifically, $A_1 = 1$ and $\log A_2 \equiv a_2 \sim N\left(-\frac{1}{2}\sigma_a^2, \sigma_a^2\right)$. We assume that a_2 is realized in period 2. But the investors, as the initial capital holders, receive advance information (news) about a_2 in period 0.[11] The workers do not have information about a_2, but they can extract some information about it from the price of capital. We start off with this simple information structure in the baseline model to highlight the key mechanism of our model. Later we will generalize the information structure.

The investors The continuum of investors receive a perfect signal about a_2 in period 0. We index investors by j for notational convenience. Investor j sells $1 - K_{j1}$ capital in period 0 and holds K_{j1} to period 1. His consumption in period 1 is hence given by

$$C_{j1} = P_0(1 - K_{j1}) + R_1 K_{j1}.$$

The investor's optimal capital holdings, K_{j1}, are given by

$$\max_{K_{j1}} \mathbb{E}[C_{j1} | \Omega_{j0}],$$

that is,

$$\max_{K_{j1}} \mathbb{E}[\,(R_1 - P_0)\,K_{j1} | \Omega_{j0}], \tag{2}$$

where $\Omega_{j0} = \{a_2, P_0\} = \Omega_0$ is the information set of investor j in period 0.

The workers We index workers by i. A worker consumes in period 2 and supplies labor to the firm in both periods 1 and 2. The workers' utility function is given by

$$U_i = C_{i2} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma} - \frac{\psi_2}{1+\gamma} N_{i2}^{1+\gamma},$$

[11] A large literature in macroeconomics has documented the importance of news shocks in explaining stock prices and business cycles (see, e.g., Beaudry and Portier (2006)).

for $\gamma > 0$. Worker i's budget constraints are

$$K_{i2} = W_1 N_{i1}, \tag{3}$$

$$C_{i2} = R_2 K_{i2} + W_2 N_{i2}. \tag{4}$$

For simplicity, we assume that the workers supply their labor inelastically in period 2, i.e., $N_{i2} = 1$. This is automatically true if we assume $\psi_2 = 0$. Allowing an elastic labor supply in period 2 complicates the algebra but does not change the model results qualitatively. Later when we study the OLG model, this assumption becomes unnecessary. Denote by $\Omega_{i1} = \{R_1, W_1, P_0\} = \Omega_1$ the information set of worker i in period 1. Using the budget constraints of (3) and (4), the worker's labor decision in period 1 is given by

$$\max_{N_{i1}} \mathbb{E}\left[R_2 W_1 N_{i1} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma} | \Omega_{i1}\right]. \tag{5}$$

Figure 1 summarizes the timeline of the model setup, where sentiment shock z will be explained shortly.

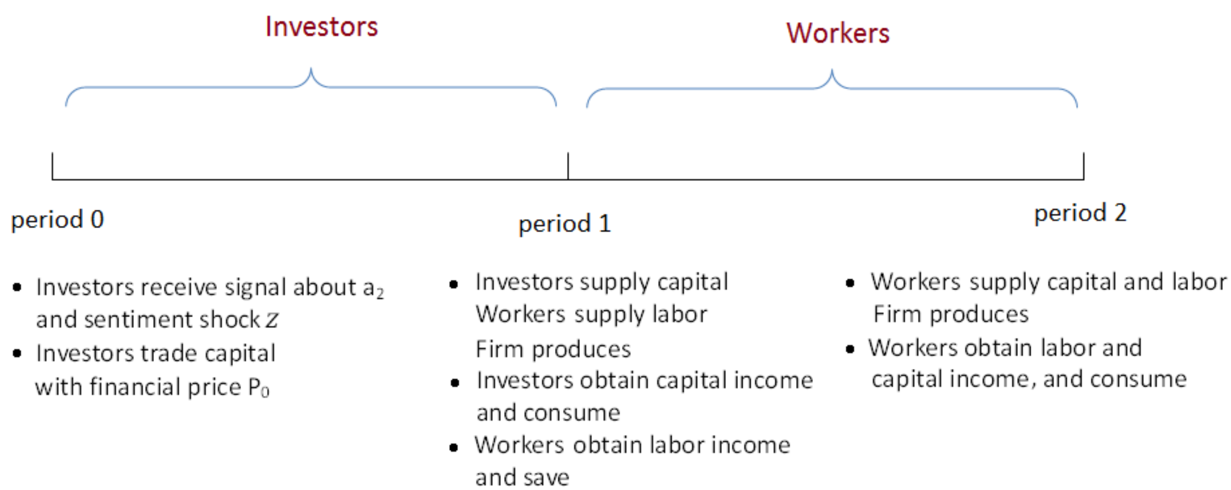

Figure 1: Timeline

The first-order condition of the investors' problem in (2) is

$$0 = \mathbb{E}[R_1 - P_0 | \Omega_0] \tag{6}$$

and the first-order condition of the workers' problem in (5) is

$$\psi N_{i1}^{\gamma} = W_1 \mathbb{E}[R_2 | \Omega_1]. \tag{7}$$

We also have

$$W_1 = (1-\alpha)A_1 K_1^{\alpha} N_1^{-\alpha},\ R_1 = \alpha A_1 K_1^{\alpha-1} N_1^{1-\alpha} \tag{8}$$

and

$$W_2 = (1-\alpha)A_2 K_2^{\alpha},\ R_2 = \alpha A_2 K_2^{\alpha-1}. \tag{9}$$

With the above first-order conditions, we are ready to define an equilibrium formally.

3 Equilibrium

Definition 1 *An equilibrium is a set of price functions $P_0 = P_0(a_2), W_1 = W_1(a_2)$, $R_1 = R_1(a_2), W_2 = W_2(a_2), R_2 = R_2(a_2)$, and the optimal capital holdings $K_{j1} = K_1(a_2, P_0)$ for the investors, and the labor choices $N_{i1} = N_1(R_1, W_1, P_0)$ for the workers and their capital holdings $K_{i2} = K_2(R_1, W_1, P_0)$ in period 2 such that: 1) Equations (6) to (9) are satisfied; 2) all markets clear*

$$\begin{aligned}
\int K_{j1} dj &= 1 \\
\int N_{i1} di &= N_1 \\
\int K_{i2} di &= K_2.
\end{aligned}$$

We are now ready to characterize the equilibrium. Noticing that the workers are homogeneous and have the same information set, i.e., $\Omega_{i1} = \Omega_1$, we focus on the symmetric equilibrium in which $N_{i1} = N_1$. Finding the equilibrium involves solving for the key endogenous variable N_1 from equation (7). So we first solve W_1 and R_2 and express them in terms of N_1. The following steps solve the equilibrium.

1. Given $K_1 = 1$ and N_1, we have

$$\begin{aligned}
R_1 &= \alpha K_1^{\alpha-1} N_1^{1-\alpha} = \alpha N_1^{1-\alpha}, \\
W_1 &= (1-\alpha) K_1^{\alpha} N_1^{-\alpha} = (1-\alpha) N_1^{-\alpha}.
\end{aligned}$$

2. The capital in period 2 is given by the labor income in period 1. Hence we have

$$K_2 = W_1 N_1 = (1-\alpha) N_1^{1-\alpha}.$$

3. We then express R_2 in terms of N_1:

$$R_2 = \alpha A_2 K_2^{\alpha-1} = \alpha A_2 \left[(1-\alpha) N_1^{1-\alpha}\right]^{\alpha-1}.$$

4. In a symmetric equilibrium where $N_{i1} = N_1$, equation (7) becomes

$$N_1^{\gamma+1-(1-\alpha)\alpha} = \psi^{-1}\alpha(1-\alpha)^{\alpha}\mathbb{E}\left[A_2|\Omega_1\right].$$

5. We normalize $\psi^{-1}\alpha(1-\alpha)^{\alpha} = 1$ and denote $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$ and thus obtain

$$N_1 = \{\mathbb{E}\left[A_2|\Omega_1\right]\}^{\theta} = \{\mathbb{E}\left[A_2|P_0\right]\}^{\theta}. \tag{10}$$

   Notice that Ω_1 is equivalent to $\{P_0\}$ as R_1 and W_1 are both functions of N_1.

6. Finally, the price P_0 should be consistent with the investors' rational expectations (equation (6)), namely,

$$P_0 = \alpha\mathbb{E}\left[N_1^{1-\alpha}|\Omega_{j0}\right] = \alpha\mathbb{E}\left[N_1^{1-\alpha}|a_2, P_0\right]. \tag{11}$$

Equations (10) and (11) are the two key equations characterizing the equilibrium. Equation (10) says that the workers' labor supply depends on their expectation of the real aggregate TFP shock, A_2. The financial market affects the real economy through the information channel as the workers try to learn A_2 from the financial price. Equation (11) states that the price of capital depends on the marginal product of capital, which in turn depends on the real economic activities – the *aggregate* labor supply N_1. The price of capital in the financial market is higher if the investors expect an increase in the real activities. Such two-way feedback can generate rich complementarities between the financial sector and the real sector and may result in multiple equilibria. Since solving for other variables such as W_1, K_2 and Y_2 is straightforward via steps 1-4, we will mainly focus on solving N_1 and P_0 in what follows. We will show three types of equilibria of the model.

Figure 2 illustrates the two-way feedback between the financial market and the real economy.

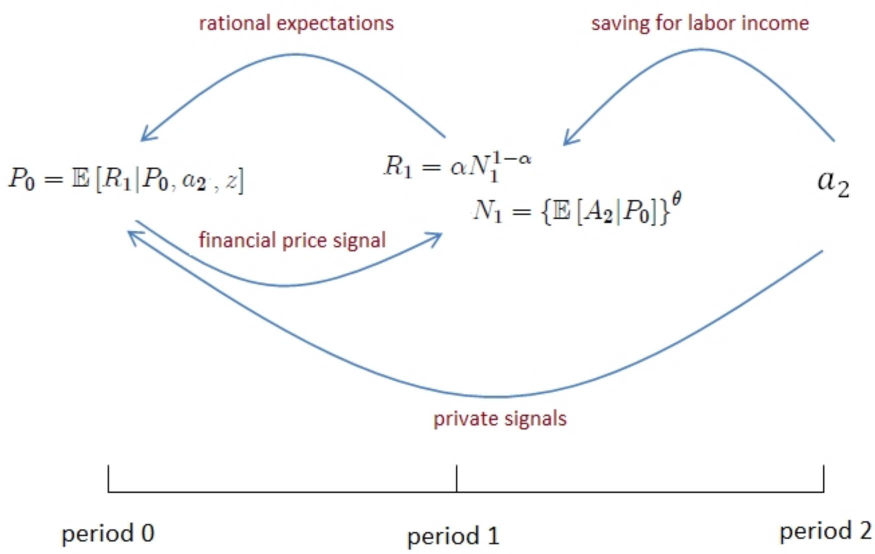

Figure 2: Two-way feedback between the financial market and the real economy

3.1 Fully-revealing Equilibrium

We first study an equilibrium where the financial price, P_0, fully reveals the fundamental uncertainty a_2. We call this equilibrium the fully-revealing rational expectations equilibrium. We have the following proposition.

Proposition 1 *There exists a fully-revealing equilibrium in which*

$$\log P_0 = \log \alpha + (1 - \alpha)\theta a_2, \tag{12}$$

and

$$\log N_1 = \theta a_2. \tag{13}$$

Proof. *The proof is straightforward. It is easy to see that equations (10) and (11) both hold.* ■

Equation (12) implies that the capital price in period 0 fully reveals a_2. This is a self-fulfilling equilibrium. If all investors believe that the dividend R_1 in the next period depends on a_2, competition in period 0 will result in that in equilibrium the price must fully reveal a_2. Since the financial price fully reveals a_2, the workers face no uncertainty in deciding their labor supply in period 1. As a result, their labor choice is $N_1 = \exp(\theta a_2)$. Since $R_1 = \alpha N_1^{1-\alpha}$, the capital dividend indeed depends on a_2 and it verifies the investors' initial beliefs. Hence, (12) and (13) constitute a rational expectations equilibrium. In this equilibrium, the financial market is informationally efficient, as the uninformed workers can learn valuable information from the informed investors through the asset price.

For the fully-revealing equilibrium, the outputs in periods 1 and 2 are

$$\log Y_1 = (1 - \alpha) \log N_1 = (1 - \alpha) \theta a_2$$

and

$$\log Y_2 = \log \{A_2 [(1 - \alpha) Y_1]^\alpha\} = a_2 + \alpha [\log(1 - \alpha) + (1 - \alpha) \theta a_2],$$

respectively.

3.2 Non-revealing Equilibrium

However, there also exists a non-revealing equilibrium where the capital price does not reveal information about a_2 at all. We characterize such an equilibrium in the following proposition.

Proposition 2 *There exists a non-revealing equilibrium in which*

$$\log P_0 = \log \alpha$$

and

$$\log N_1 = 0.$$

Proof. *The proof is straightforward and hence omitted.* ■

If investors in period 0 think that the workers' labor supply is $N_1 = 1$ and hence the dividend per unit capital $R_1 = \alpha N_1^{1-\alpha}$ is independent of a_2, then their advance information about a_2 becomes irrelevant. The competition in the financial market then drives $P_0 = R_1 = \alpha N_1^{1-\alpha} = \alpha$. Under such a price the workers can learn nothing about a_2 from the capital price, and thus by equation (10) their labor supply is determined by the unconditional mean of A_2, which by our assumption is one. Hence, $N_1 = 1$ or $\log N_1 = 0$. Again, the investors' initial belief that $N_1 = 1$ is verified.

For the non-revealing equilibrium, the outputs in periods 1 and 2 are

$$\log Y_1 = (1-\alpha)\log N_1 = 0$$

and

$$\log Y_2 = \log\left\{A_2\left[(1-\alpha)\,Y_1\right]^{\alpha}\right\} = a_2 + \alpha\log(1-\alpha),$$

respectively.

3.3 Sentiment-driven Fluctuations

We now show that there are other types of equilibria in our model. We call them sentiment-driven equilibria. Suppose that the investors in the financial market also observe a non-fundamental shock, $z \sim N(0, \sigma_z^2)$, which is affected by their sentiment or psychology. We assume that z and a_2 are independent. That is, the information set in period 0 becomes $\Omega_{j0} = \{P_0, a_2, z\} = \Omega_0$. We are interested in the equilibrium in which the aggregate labor supply in period 1 takes the form $\log N_1 = \bar{n} + \phi a_2 + z$, where $\bar{n}$ and ϕ are coefficients to be determined. That is, in such an equilibrium sentiments matter. We have the following proposition.

Proposition 3 *There exists a continuum of equilibria indexed by $0 \le \sigma_z^2 \le \frac{\theta^2}{4}\sigma_a^2$, in which the price P_0 is given by*

$$\log P_0 = \bar{p} + \log\alpha + (1-\alpha)\left(\phi a_2 + z\right) \tag{14}$$

and

$$\log N_1 = \bar{n} + \phi a_2 + z, \tag{15}$$

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2 \sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha} \tag{16}$$

and $\bar{p} = \bar{n} = 0$.

Proof. *See Appendix.* ■

When investors perceive that $\log N_1 = \phi a_2 + z$, they believe that the dividend per unit capital, R_1, is affected not only by the fundamental shock a_2 but also by the sentiment z. Competition in the financial market in period 0 will then drive the price to $P_0 = R_1 = \alpha N_1^{1-\alpha}$. With this price, investors are happy to trade. However, for the workers, the price P_0 now only partially reveals the fundamental shock a_2. The workers face a signal extraction problem – using the price to forecast a_2. The actual labor supply will then be a function of P_0. The size of the fundamental shock relative to the sentiment shock has to satisfy some restrictions so that the actual labor supply of the workers is exactly the same as investors think it would be. This explains condition (16). When condition (16) holds, the initial belief of the investors that $\log N_1 = \phi a_2 + z$ is verified. To see this, by (10), the actual labor supply of the workers is given by $\log N_1 = \theta \left\{ \mathbb{E}\left[a_2 | \phi a_2 + z\right] + \frac{1}{2} var(a_2 | \phi a_2 + z) \right\}$, which can be calculated as $\log N_1 = \frac{\theta \phi \sigma_a^2}{\phi^2 \sigma_a^2 + \sigma_z^2} (\phi a_2 + z)$. Note that $\frac{\theta \phi \sigma_a^2}{\phi^2 \sigma_a^2 + \sigma_z^2} = 1$ by rearranging (16). Therefore, P_0 and N_1 as defined in equations (14) and (15) indeed constitute a rational expectations equilibrium.

For the sentiment-driven equilibria, the outputs in periods 1 and 2 are, respectively,

$$\log Y_1 = (1 - \alpha) \log N_1 = (1 - \alpha)(\phi a_2 + z)$$

and

$$\log Y_2 = \log \left\{ A_2 \left[(1 - \alpha) Y_1 \right]^\alpha \right\} = a_2 + \alpha \left[\log(1 - \alpha) + (1 - \alpha)(\phi a_2 + z) \right].$$

The sentiment-driven fluctuation studied in this subsection links the Keynesian notions of "beauty contests" and "animal spirits". What matters to an individual investor is not his own assessment of the fundamental a_2 and thereby its impact on dividend R_1, but his conjecture about the actions of other investors, as in a standard beauty contest game. This comes about because of the feedback from the first stage (period 0) to the second stage (period 1), which generates endogenous complementarities between actions of investors. At the same time, the sentiment shocks in the financial market affect the real economy through the asset price and generate fluctuations in aggregate output as if they were driven by "animal spirits". A long tradition in macroeconomics has resorted to models that feature multiple equilibria to explain the observed "animal spirits"-styled fluctuations. In our model, sentiment-based asset prices with informational feedback drive multiple equilibria.[12,13]

Three remarks on the sentiment-driven equilibria are in order. First, in our model, the capital price P_0 is always equal to the dividend R_1 in period 1 (no arbitrage). Nevertheless, sentiment-driven equilibria exist because sentiments can endogenously drive the dividend R_1 in period 1.[14] Figure 3 illustrates the continuum of sentiment-driven equilibria. The fully-revealing equilibrium and the non-revealing equilibrium analyzed in the previous two subsections are two special cases of the sentiment-driven equilibria. Note that sentiment-driven equilibria exist for $\sigma_z \in (0, \frac{\theta}{2}\sigma_a]$, which means that sentiment-driven equilibria are more likely when σ_a is higher.

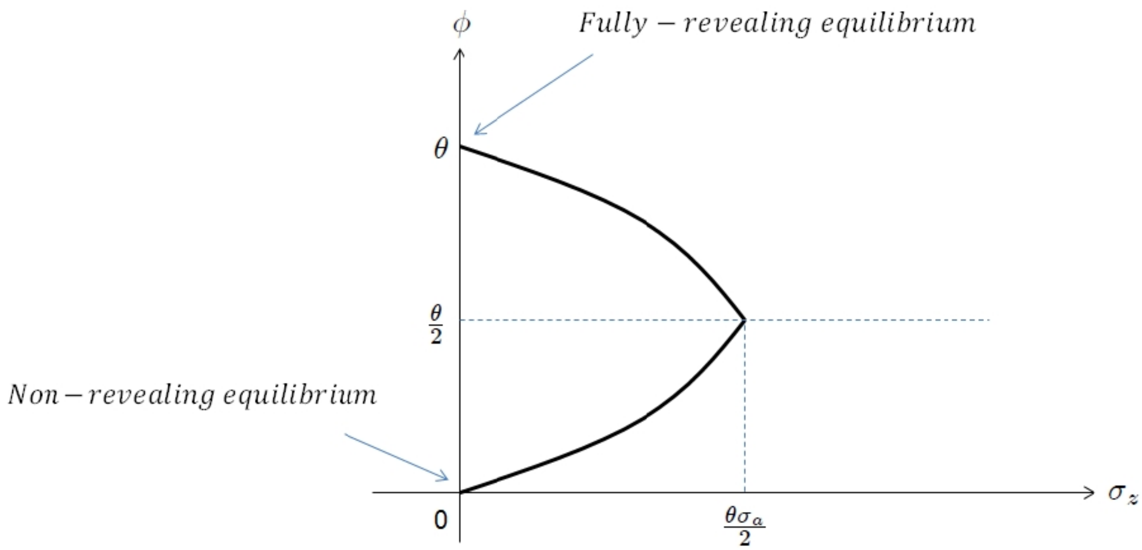

Figure 3: A continuum of self-fulfilling sentiment-driven equilibria

Second, as in the finance literature, we can use $\frac{1}{Var(a_2|P_0)}$, the reciprocal of the variance of a_2 conditional on the price, to measure the price informativeness about a_2. It is easy to calculate

$$\frac{1}{Var(a_2|\phi a_2 + z)} = \frac{\theta}{\theta - \phi}\frac{1}{\sigma_a^2},$$

[12] In our model setup with asymmetric periods of consumption (of investors and workers), there are two frictions: the limited participation friction as in a typical OLG model and the informational friction. If we focus on the second friction only, we are able to prove that the second-best constrained efficiency corresponds to the fully-revealing equilibrium and the sentiment-driven equilibria are welfare reducing, which gives the welfare implication of the sentiment-driven fluctuations.

[13] In our model with a competitive financial market, the investors are price-takers and the collusion among them is precluded. Empirically, in a large financial market, collusion may be difficult. Nonetheless, for theoretical models of collusion in financial markets on individual stocks, see Allen and Gale (1992) and also Peck (2014). For recent work related to ours that also excludes collusion see, for example, Hellwig and Veldkamp (2009), Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren and Yuan (2013).

[14] Sentiment-driven equilibria would not be possible if the labor supply is assumed to be constant or exogenously given. To see this, let us assume that $N_1 = 1$ without loss of generality. Equation (11) then immediately implies a unique price $P_0 = \alpha$. This differs from Benhabib and Wang (2013), where the sentiment can drive the asset price to diverge from its underlying dividend.

which is monotonically increasing in ϕ. Hence, if the continuum of equilibria in Proposition 3 (Figure 3) are indexed by $\phi \in [0, \theta]$, we can rank them in terms of price informativeness.

Third, it is worth noting that trading based on sentiments in our model is different from noise trading in at least two aspects. i) Noise traders are irrational and unaware of their mistakes. In contrast, the investors in our model are fully rational. Although they are aware that a sentiment shock is non-fundamental, it is optimal for them to trade on the shock if their peers choose to do so. ii) Noise trading volatility is exogenous and can be arbitrary. By contrast, sentiment volatility in our model has an endogenous upper limit, i.e., $\sigma_z^2 \leq \frac{\theta^2}{4}\sigma_a^2$ as shown in Proposition 3.

Role of the assumptions Here we discuss two assumptions in our model. First, informed investors are short-lived and they have long-lived information. This assumption is made mainly for the purpose of introducing the OLG model later in a consistent setup.[15] Allen, Morris and Shin (2006) provide detailed explanations and motivation for this assumption. In the context of our model, the reason for making this assumption is even stronger. The investors are the initial capital holders; naturally they may have some advance information about a_2. More importantly, financial markets can aggregate the dispersed information of investors (Grossman and Stiglitz (1980)). We will show in the next section that the investors in our model do not need to have perfect information or an information advantage over the workers. Second, we assume a Cobb-Douglas production function under which capital and labor are complementary in production in period 1. Our model mechanism is robust to the setting with a general CES production function.[16]

4 Generalized Information Structures

In the baseline model, we assumed a simple information structure to highlight the core mechanism of our model. In this section, we generalize our information structure along several dimensions and show that our result is robust.

[15] For our baseline model, we can instead assume that investors are long-lived and their consumption occurs in period 2. This would not change the result of our model under the reasonable assumption that limited commitment frictions prevent trades between the investors and the workers before workers make their labor decision and obtain labor income in period 1. Note that investors' capital in period 0 fully depreciates in period 1 and is not carried over to period 2. Formally, if investors consume in period 2, an investor's optimal capital trading decision in period 0 is given by $\max_{K_{j1}} \mathbb{E}[C_{j2}|\Omega_{j0}]$, where $C_{j2} = R_2 C_{j1}$, $C_{j1} = P_0(1 - K_{j1}) + R_1 K_{j1}$ and $R_2 = \alpha A_2 K_2^{\alpha-1} = \alpha A_2 \left[N_1^{1-\alpha}\right]^{\alpha-1}$ by noting $K_2 = Y_1 = N_1^{1-\alpha}$; we "misuse" notation and use C_{j1} to denote savings carried over to period 2 for expositional simplicity. It is easy to show that the above optimization problem is equivalent to $\max_{K_{j1}} \mathbb{E}[C_{j1}|\Omega_{j0}]$.

[16] Under a general CES production function, the marginal product of capital and thus its return are still increasing in labor. So our result of the existence of multiple self-fulfilling equilibria does not change.

4.1 Dispersed Information and the Implementation of Equilibria

So far, we have assumed that investors have perfect information about fundamental shock a_2 and sentiment shock z. We now allow for dispersed information. Specifically, the information set of investor j is assumed to be $\Omega_{j0} = \{P_0, a_2+\varepsilon_j, z+\delta_j\}$, where $a_2+\varepsilon_j$ is his private signal about a_2 and $z+\delta_j$ is his private signal about sentiment z. It is also assumed that $\varepsilon_j \sim N(0, \sigma_\varepsilon^2)$, $\delta_j \sim N(0, \sigma_\delta^2)$, $cov(\varepsilon_j, \delta_j) = 0$, and that ε_j as well as δ_j is independent across investors.

Under this setup, we can immediately conclude that Propositions 1 and 3 still hold based on the concept of rational expectations equilibrium (REE).[17] In fact, under the "efficient" market price given in (14) that already fully reflects $\phi a_2 + z$, any noise (i.e., ε_j and δ_j) on top of a_2 and z have no value in inferring $R_1 = \alpha N_1^{1-\alpha}$ when N_1 is given by (15). That is, when investors trade capital in the financial market, they do not need to condition their decisions on their private information as the market price is informative enough compared with their private signal.

The question, however, is where the "efficient" price comes from in the first place. Such REEs raise a natural question about equilibrium implementability (see, e.g., Vives (2014) for more discussions). Traders do not condition their demand and supply on their private information, yet the market price somehow magically aggregates their private information and becomes efficient. In what follows, we study the implementation of the sentiment-driven equilibrium as well as the fully-revealing equilibrium under dispersed information of investors.

We adopt the approach proposed by Vives (2014) (also used in Benhabib and Wang (2013)). Unlike in a classic REE trading game where a trader submits his demand conditional on both the market price and his private signal, in Vives' trading game, a trader submits his *demand function (schedule)* conditional on his private signal only, and then the market auctioneer collects the demand schedules from all traders and sets a price that can clear the market.

To tailor to the setting of private valuations in Vives (2014), we make an additional but weak assumption. Beside the dividend R_1, a scrap value or private benefit in proportion to R_1 is derived when investors hold capital to period 1, in the spirit of Holmstrom and Tirole (1997). Specifically, the gain per unit of capital that investor j obtains by holding that capital to period 1 is $R_1 e^{u_j}$, where $u_j \sim N\left(-\frac{1}{2}\sigma_u^2, \sigma_u^2\right)$ measures the heterogeneity across investors in private benefit of holding capital. Investor j receives private signals $s_{j0} = a_2 + u_j + \varepsilon_j$ and $l_{j0} = z + \delta_j$ in period 0; that is, $\Omega_{j0} = \{a_2 + u_j + \varepsilon_j, z + \delta_j\}$. In his eyes, the fundamental value of the capital includes not only a_2 but also u_j, so it is natural to assume that his private signal s_{j0} concerns $a_2 + u_j$ and not a_2.

For simplicity, we also assume that the *net* demand for capital for an investor is limited up to

[17] The non-revealing equilibrium in Proposition 2 naturally holds. We are interested in the fully-revealing equilibrium and the sentiment-driven equilibrium where the capital price efficiently aggregates the dispersed information of investors.

$\bar{d} > 0$ and short sales are not allowed. Denote the demand schedule by $d_j(p_0; s_{j0}, l_{j0})$, meaning that investor $j's$ submitted demand function is $d_j(p_0)$ conditional on his private signals being s_{j0} and l_{j0}, where $p_0 \equiv \log P_0$. We have the following proposition.

Proposition 4 *There exists a continuum of self-fulfilling equilibria indexed by* $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4}\sigma_a^2$*, in which the demand schedule of investors is*

$$d_j(p_0; s_{j0}, l_{j0}) = \begin{cases} \bar{d} & \text{if } \phi s_{j0} + l_{j0} \geq \frac{p_0 - (u^* + \log\alpha)}{1-\alpha} + \phi \frac{\sigma_u^2 + \frac{1}{\phi^2}\sigma_\delta^2 + \sigma_\varepsilon^2}{\sigma_u^2} u^* \\ -1 & \text{otherwise} \end{cases} \tag{17}$$

and the equilibrium price is given by

$$p_0 = u^* + \log\alpha + (1-\alpha)(\phi a_2 + z)$$

and

$$\log N_1 = \phi a_2 + z$$

$$\log R_1 = p_0 - u^* = \log\alpha + (1-\alpha)(\phi a_2 + z),$$

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha} \quad \text{and} \quad u^* = \frac{\Phi^{-1}\left(\frac{\bar{d}}{1+\bar{d}}\right)\sigma_u^2}{\sqrt{\sigma_u^2 + \sigma_\varepsilon^2 + \frac{\sigma_\delta^2}{\phi^2}}}.$$

Proof. *See Appendix.* ■

The equilibrium outcome in Proposition 4 is essentially the same as that in Proposition 3. The difference is that now there is an explicit mechanism for aggregating the dispersed information of investors into the asset price, which in turn influences the decision of workers. The equilibria are still self-fulfilling. Depending on investors' expectation of p_0 (i.e., the extent to which the market price will reflect fundamentals versus sentiments), they will choose different demand schedules, which will in turn result in different market clearing prices. Sentiments endogenously drive the dividend R_1 in period 1. Note that when $\sigma_z^2 = 0$, $\sigma_\delta^2 = 0$ and $l_{j0} = 0$, the equilibrium corresponds to the fully-revealing equilibrium about a_2. In other words, the fully-revealing equilibrium is a special case of the sentiment-driven equilibrium.

Since we have analyzed the REE implementation under dispersed information, in the following sections we will apply the concept of REEs directly if investors or workers have dispersed information.

4.2 Private Information of Workers

In this subsection, we assume that not only investors but also workers receive private information about the TFP shock and the sentiment shock. We show that our result of sentiment-driven equilibria is robust to the alternative information structures under REEs. For expositional convenience, we repeat the equilibrium conditions of (10) and (11) here:

$$N_1 = \{\mathbb{E}[A_2|\Omega_{i1}]\}^{\theta} \tag{18}$$

$$P_0 = \alpha \mathbb{E}\left[N_1^{1-\alpha}|\Omega_{j0}\right]. \tag{19}$$

First, we assume that not only investors but also workers receive signals about A_2. Specifically, we assume that the information set for investors in period 0 is $\Omega_{j0} = \{P_0, a_2^I + \varepsilon_j,\ z + \delta_j\}$ and the information set for workers in period 1 is $\Omega_{i1} = \{R_1, W_1, P_0, a_2^H + v_i\}$, where $s_{j0} = a_2^I + \varepsilon_j$ is the private signal about a_2 received by investor j in period 0 and $s_{i1} = a_2^H + v_i$ is the private signal about a_2 received by worker i in period 1. We assume that $cov(a_2, a_2^I) > 0$, $cov(a_2, a_2^H) > 0$ and $cov(a_2^I, a_2^H) = 0$. For instance, $a_2 = \omega a_2^I + (1-\omega) a_2^H$, with $0 < \omega < 1$ and $cov(a_2^I, a_2^H) = 0$, satisfies the assumptions. Without loss of generality, we assume that $a_2 = a_2^I + a_2^H$. In addition, we assume the unconditional distributions to be $a_2^I \sim N(-\frac{1}{2}\sigma_I^2, \sigma_I^2)$ and $a_2^H \sim N(-\frac{1}{2}\sigma_H^2, \sigma_H^2)$.

Under the above alternative information structure, we have the following proposition.

Proposition 5 *There exists a continuum of sunspot equilibria indexed by $0 \le \sigma_z^2 \le \frac{\theta^2}{4}\sigma_I^2$, in which the price P_0 is given by*

$$\log P_0 = \log \alpha + (1-\alpha)\left(\phi a_2^I + z\right), \tag{20}$$

and

$$\log W_1 = \log(1-\alpha) - \alpha\left[\left(\phi a_2^I + z\right) + \theta a_2^H\right], \tag{21}$$

$$\log N_1 = \left(\phi a_2^I + z\right) + \theta a_2^H, \tag{22}$$

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_I^2 - 4\sigma_z^2}}{2\sigma_I}. \tag{23}$$

Proof. *See Appendix.* ■

The intuition behind Proposition 5 is similar to that behind Proposition 3. When workers decide on their labor supply, they need to forecast $a_2 = a_2^I + a_2^H$. They can infer a_2 from three pieces of information: financial price P_0, wage W_1 and their own signal $s_{i1} = a_2^H + v_i$. Wage W_1 efficiently aggregates all private signals, s_{i1}, to clear the labor market. This can be understood by noting that the total labor demand is $N_1^d = \left(\frac{W_1}{1-\alpha}\right)^{-\frac{1}{\alpha}}$, which only depends on the wage. The worker labor supply is characterized by some function N_1^s such that $N_{i1}^s = N_1^s(W_1, a_2^H + v_i, P_0)$. The market clearing condition requires $N_1^d = \int N_1^s(W_1, a_2^H + v_i, P_0)di$, which means $W_1 = W(P_0, a_2^H)$ for any function N_1^s. Since workers know P_0, they can infer a_2^H perfectly from W_1.

We can further generalize the information structure by allowing investors' and workers' signals about A_2 to be correlated. In addition, we can allow workers to also receive some information about sunspots and their signals on sunspots to be correlated with those of investors. Specifically we assume that $a_2 = a_2^I + d + a_2^H$, where d is a random variable independent of a_2 and $d \sim N(-\frac{1}{2}\sigma_d^2, \sigma_d^2)$; a_2^I and a_2^H have the unconditional distributions as previously specified with $cov(a_2^I, a_2^H) = 0$. Similarly, we assume that $\frac{z}{\sigma_z} = z^I + \chi + z^H$, where z^I, χ and z^H all have the standard normal unconditional distribution, and $cov(z, \chi) = 0$ and $cov(z^I, z^H) = 0$. The information set for investors is assumed to be $\Omega_{j0} = \{P_0, a_2^I + d + \varepsilon_j,\ z^I + \chi + \delta_j, d, \chi\}$ and the information set for workers is $\Omega_{i1} = \{R_1, W_1, P_0, a_2^H + d + v_i, z^H + \chi + \varsigma_i, d, \chi\}$, where $\varsigma_i \sim N(0, \sigma_\varsigma^2)$. In other words, the signals about both a_2 and z are correlated across investors and workers, where d and χ represent common information for investors and workers. Under this alternative information structure, we show that there exists a continuum of sunspot equilibria indexed by $0 \le \sigma_z^2 \le \frac{\theta^2}{4}\sigma_I^2$, with

$$\log P_0 = \log \alpha + (1-\alpha)\left(\phi a_2^I + \sigma_z z^I + \theta d\right),$$

$$\log W_1 = \log(1-\alpha) - \alpha\left[\left(\phi a_2^I + \sigma_z z^I + \theta d\right) + \theta a_2^H\right],$$

$$\log N_1 = \left(\phi a_2^I + \sigma_z z^I + \theta d\right) + \theta a_2^H,$$

where ϕ is given by

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_I^2 - 4\sigma_z^2}}{2\sigma_I}.$$

The proof is very similar to that of Proposition 5 and hence is omitted. A conclusion we can draw is that as long as there is informational segmentation between investors and workers, but not necessarily asymmetry (i.e., the investors need not have an information advantage over the workers), there exist sunspot equilibria.[18] So without loss of generality, in the following sections, we assume that only the investors have information about the fundamental shock A_2 and the sunspot z.

5 More implications

5.1 Non-linear Asymmetric Equilibria

So far we have focused on symmetric equilibrium for the sake of tractability. We now consider the possibility of asymmetric equilibria, which can be appealing for example if prices are generally more

[18] We owe this summary to the referee.

informative when fundamentals are strong. Also, asset price collapses can sometimes occur with a small change in economic fundamentals (see, e.g., the evidence documented by Culter, Poterba, and Summers (1989)). Several studies (e.g., Yuan (2005) and Barlevy and Veronesi (2003)) have attempted to explain this asymmetric and nonlinear response in asset prices in a framework of informational frictions in an exchange economy. We now show that our sentiment-driven equilibria can also generate such asymmetric responses in asset prices in a production economy.

To illustrate the intuition, we first consider a simple case. We conjecture that the equilibrium price takes the following form:

$$\log P_0 = \begin{cases} \log \alpha + (1-\alpha)\theta a_2 & \text{if } a_2 \geq 0 \\ \log \alpha + (1-\alpha)\theta \log \left[\frac{\Phi(-\frac{1}{2}\sigma_a)}{\Phi(\frac{1}{2}\sigma_a)} \right] & \text{if } a_2 < 0 \end{cases}, \tag{24}$$

where $\Phi(\cdot)$ denotes the c.d.f. of the standard normal distribution. As $\frac{\Phi(-\frac{1}{2}\sigma_a)}{\Phi(\frac{1}{2}\sigma_a)} < 1$, it follows that $\log P_0 < \log \alpha$ iff $a_2 < 0$. So, from the price $\log P_0$ given by (24), the workers can infer perfectly whether $a_2 < 0$ or not. We now verify that the price given by (24) indeed forms a rational expectation equilibrium. After observing the price $\log P_0 \geq \log \alpha$, the workers can infer that $a_2 = (\log P_0 - \log \alpha) / ((1-\alpha)\theta)$. In this case, the workers face no uncertainty and hence their labor supply is $\log N_1 = \theta a_2$ according to equation (10). By (11), the asset price is hence given by (24). On the other hand, if the workers see that the financial price is below $\log \alpha$, they know for sure that $a_2 < 0$. Their rational expectation of A_2 is thus $\mathbb{E}[A_2|P_0] = \mathbb{E}[A_2|A_2 < 1] = \frac{\Phi(-\frac{1}{2}\sigma_a)}{\Phi(\frac{1}{2}\sigma_a)}$ by the property of lognormal distribution. So their labor supply is $N_1 = \left[\frac{\Phi(-\frac{1}{2}\sigma_a)}{\Phi(\frac{1}{2}\sigma_a)} \right]^\theta$ according to (10). By (11), the asset price is hence verified to be (24). To summarize, when $a_2 \geq 0$ the equilibrium is fully revealing; when $a_2 < 0$ the equilibrium is non-revealing.[19]

Despite its simplicity, the above example provides two general insights. First, by construction the price is more informative when the fundamentals are strong. Second, the price is discontinuous in the fundamental value measured by a_2. There is a discrete jump around $a_2 = 0$. That is, a large fall in the asset price occurs with a small decrease in a_2 around $a_2 = 0$ if $(1-\alpha)\theta$ or σ_a is big.

Now we consider non-linear asymmetric sentiment-driven equilibria. Again, we are interested in an equilibrium with

$$\log P_0 = \begin{cases} \log \alpha + (1-\alpha)\theta a_2 & \text{if } a_2 \geq 0 \\ \log \alpha + p(a_2, z) & \text{if } a_2 < 0 \end{cases}, \tag{25}$$

where z is the sentiment and $p(a_2, z)$ is some nonlinear function. The difficulty of constructing such an equilibrium lies in that both the distribution function of z and the price function $p(\cdot)$ have to be consistent with rational expectations. Denote by $f(z)$ the density function of z. Conditions

[19] That investors believe that the real side of the economy is asymmetrically affected by fundamentals a_2 when $a_2 < 0$ and $a_2 \geq 0$ can be for reasons suggested by Jin and Myers (2006) and Bleck and Liu (2007).

(10) and (11) imply that a rational expectation equilibrium must satisfy

$$p(a_2, z) = (1-\alpha)\theta \log \mathbb{E}[\exp(a_2)|p(a_2, z)] < 0, \tag{26}$$

for all $a_2 < 0$ and all possible z. Since the distribution of z can in principle be arbitrary, it is impossible to give a complete characterization of all nonlinear asymmetric equilibria. And in general, the equilibrium price function will not permit a closed-form solution. Hence, we focus on a special case of the equilibrium which yields closed-form solutions for tractability. Suppose that z is distributed with the following density function

$$f(z) = \begin{cases} 0 & \text{if } z \geq 0 \\ \frac{1}{\Phi(\frac{1}{2}\sigma_a)} \cdot \frac{1}{\sigma_a\sqrt{2\pi}} e^{-\frac{(z+\sigma_a^2/2)^2}{2\sigma_a^2}} & \text{if } z < 0 \end{cases}, \tag{27}$$

that is, z follows the *truncated* normal distribution of $z \sim N\left(-\frac{1}{2}\sigma_a^2, \sigma_a^2\right)$ with $z < 0$. In other words, a_2 and z are independently and identically distributed conditional on $a_2 < 0$. We have the following sentiment-driven equilibrium.

Proposition 6 *Suppose that z has density function given by (27). There exists an asymmetric sentiment-driven equilibrium in which the price takes the form given by (25) with*

$$p(a_2, z) = (1-\alpha)\theta \log \left[\frac{\exp(a_2) + \exp(z)}{2}\right]. \tag{28}$$

Proof. *See Appendix.* ■

Since $\exp(z) < 1$ with probability 1, we have $p(a_2 = 0^-, z) = (1-\alpha)\theta \log\left[\frac{1+\exp(z)}{2}\right] < 0$ with probability 1. This means that when fundamental a_2 declines from positive to negative, there is a downward jump in the asset price with probability 1. More importantly, in the limit, the decline is purely driven by the sentiment shock if the realized a_2 happens to be 0^-. An arbitrarily small deterioration in economic fundamentals can generate a large crisis in the asset price and real output under a very pessimistic sentiment shock z.

It is easy to show that there are an infinite number of non-linear asymmetric sentiment-driven equilibria. In fact, we can assume that $z = (z_1, z_2, ...z_\Gamma)$ and z_κ is i.i.d. with density function (27), where $\kappa = 1, 2,...\ \Gamma$. So there are an infinite number of asymmetric sentiment-driven equilibria indexed by Γ, where

$$p(a_2, z) = (1-\alpha)\theta \log \left[\frac{\exp(a_2) + \sum_{\kappa=1}^{\Gamma} \exp(z_\kappa)}{\Gamma + 1} \right]. \tag{29}$$

Cleary, when $\Gamma = 0$, the equilibrium corresponds to the fully-revealing equilibrium in Proposition 1; when $\Gamma = +\infty$, the equilibrium corresponds to the non-revealing equilibrium for $a_2 < 0$, as in (24).

5.2 Contagion and Comovement

A large empirical literature has documented contagion in asset price movements (see Yuan (2005) for a discussion of the findings in the literature). In particular, empirical evidence suggests that contagion cannot be explained by fundamentals and is asymmetrical in market downturns and upturns (see, e.g., Karolyi and Stultz (1996), King and Wadhwani (2000), Ang and Chen (2002), Connolly and Wang (2003)). In attempting to explain the international synchronization of business cycles during financial crises, while the theory of the credit channel proposed by Perri and Quadrini (2013) is useful in explaining such synchronization among industrial countries, it alone cannot explain why many emerging countries with heavy capital controls can also fall into a deep recession (Chudik and Fratzscher (2012)). In this subsection, we use our sentiment-driven equilibrium (informational channel) to illustrate economic contagion and comovement.

We now extend the model economy to a continuum of countries/markets indexed by ℓ. For simplicity, we assume that these countries are in autarky and there are no trade or financial linkages. The productivity shock in country ℓ in period 2 is given by

$$\log A_{2\ell} = g + a_{2\ell} \equiv \tilde{a}_{2\ell},$$

where $g \sim N\left(-\frac{1}{2}\sigma_g^2, \sigma_g^2\right)$ is a global shock that affects all country and $a_{2\ell} \sim N\left(-\frac{1}{2}\sigma_a^2, \sigma_a^2\right)$ is the i.i.d. country-specific technology shock. This setup with the assumption of an exogenous global shock can be regarded as a reduced-form model of the two-country model with endogenous explicit trade linkage in our working paper version of Benhabib, Liu and Wang (2014).

We assume that country 0 (for example, the U.S.) is special in that its financial price may influence actions of the investors in other countries. This is to capture the initial trigger of the economic contagion. For simplicity, again we assume that the investors in country ℓ know g and

$a_{2\ell}$ perfectly in period 0. So the equilibrium in each country can be characterized by

$$N_{1\ell} = \left\{\mathbb{E}\left[A_{2\ell}|P_{0\ell}, P_{00}\right]\right\}^{\theta}$$

and

$$P_{0\ell} = \alpha\mathbb{E}\left[N_1^{1-\alpha}|g, a_{2\ell}, P_{0\ell}, P_{00}\right].$$

It follows immediately that the fully-revealing equilibrium in which $\log P_{0\ell} = \log\alpha + (1 - \alpha)\theta(g + a_{2\ell})$ and $\log N_{1\ell} = \theta(a_{2\ell} + g)$ constitute a rational expectation equilibrium in the open economy. In this equilibrium, contagion is purely driven by the global shock g. We now show that an informational contagion can occur, where contagion means that a pure idiosyncratic shock in country 0 can generate a synchronization of asset prices and real outputs across countries.

We have the following equilibrium that exhibits contagion through idiosyncratic technology shocks. For $\ell = 0$,

$$\begin{aligned}\log P_{00} &= \log\alpha + (1-\alpha)\theta(g + a_{20}),\\ \log N_{10} &= \theta(g + a_{20}),\end{aligned}$$

and for $\ell > 0$,

$$\begin{aligned}\log P_{0\ell} &= \log\alpha + (1-\alpha)\left\{(\phi a_{2\ell} + z_\ell) + \theta\left[\frac{\sigma_g^2}{\sigma_a^2+\sigma_g^2}(g + a_{20}) + \frac{1}{2}\frac{\sigma_a^2\sigma_g^2}{\sigma_a^2+\sigma_g^2}\right]\right\}\\ \log N_{1\ell} &= (\phi a_{2\ell} + z_\ell) + \theta\left[\frac{\sigma_g^2}{\sigma_a^2+\sigma_g^2}(g + a_{20}) + \frac{1}{2}\frac{\sigma_a^2\sigma_g^2}{\sigma_a^2+\sigma_g^2}\right],\end{aligned} \tag{30}$$

with $\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha}$, where $z_\ell \sim N\left(0, \sigma_z^2\right)$ is the sentiment shock in country $\ell > 0$. In the above equilibrium, the workers in country $\ell > 0$ infer the global shock g from price P_{00} and the local shock $a_{2\ell}$ from price $P_{0\ell}$ (together with P_{00}). In other words, local price $P_{0\ell}$ does not provide any additional information regarding g beyond what P_{00} provides; price $P_{0\ell}$ only provides noisy signals (with sentiment noise) about $a_{2\ell}$.

In the extreme case of $\sigma_z = 0$ and $\phi = 0$, we have $\log P_{0\ell} = \log\alpha + (1-\alpha)\theta\left[\frac{\sigma_g^2}{\sigma_a^2+\sigma_g^2}(g + a_{20}) + \frac{1}{2}\frac{\sigma_a^2\sigma_g^2}{\sigma_a^2+\sigma_g^2}\right]$ and $\log N_{1\ell} = \theta\left[\frac{\sigma_g^2}{\sigma_a^2+\sigma_g^2}(g + a_{20}) + \frac{1}{2}\frac{\sigma_a^2\sigma_g^2}{\sigma_a^2+\sigma_g^2}\right]$ for $\ell > 0$, that is, local price $P_{0\ell}$ becomes completely uninformative and country 0 and country ℓ are perfectly synchronized. Country 0's idiosyncratic technology shock, a_{20}, affects other countries. The intuition behind the above equilibrium is the following. In the open economy, the asset price in country 0 essentially becomes another sunspot for investors in country ℓ, in addition to their country-specific sentiment shock. In the above equilibrium, investors in country $\ell > 0$ "overreact" to the asset price in country 0. Investors in all other countries believe that workers in their country think recessions in the U.S. will have a global impact. With this belief, investors in country j will ignore local shock $a_{2\ell}$ and try to short capital when they see a fall in the asset price in the U.S. caused by idiosyncratic shock a_{20}. They will therefore push down the capital price in country j, as these investors are afraid that workers in their country will reduce their labor supply and thus lower the return on capital. When the workers see a fall in capital prices $P_{0\ell}$ and P_{00}, they will partially attribute the fall in the asset prices to a global shocks g, leading them to reduce the actual labor supply. So investors' perception of synchronization leads to actual synchronization.

In Appendix B, we also provide an asymmetric sentiment-driven equilibrium that exhibits contagion.

6 The OLG Model

In this section, we extend our baseline model to the OLG framework. We show that our sentiment-driven equilibria are robust to a dynamic setting, and we derive additional economic insights. First, the OLG model provides a dynamic equilibrium setting to study the process of saving and capital accumulation. As in the baseline model, we study the sentiment-driven rational expectations equilibria. The difference lies in the dynamic setting where the current savings of workers become the capital stock in the subsequent period. The capital stock therefore is dynamically linked across periods through savings. Its accumulation, as well as output and employment, is therefore driven not only by the private future productivity signals received by investors, but also by their sentiments. Second, the OLG model delivers novel implications for asset prices over the business cycles.

Timeline In each period t, there are five stages:

Stage 1: (Information) The old generation of workers become investors (capitalists) and a new generation of workers are born. Both capitalists and workers know the history $A^{t-1} = \{A_\tau\}_{\tau=0}^{t-1}$ and the current-period A_t. Only capitalists receive private signals about A_{t+1} to be realized in the next period.

Stage 2: (Capital trading among capitalists) Capitalists trade capital among themselves in a financial market before production based on the historical information A^t, their private signals about A_{t+1}, their private signals about sentiment shock z_t, and the capital price P_t. The sentiment shock has prior distribution $z_t \sim N(0, \sigma_z^2)$ and is i.i.d. across time.

Stage 3: (Production) Based on their capital stock, wage W_t and productivity A_t, capitalists hire workers and produce. By inferring information about A_{t+1} from prices P_t and W_t, workers decide on their labor supply.

Stage 4: (Consumption) Capitalists obtain their production revenue net of labor cost, clear the balance (if any) of capital trade in the current period and the balance (if any) of bond trade in the previous period, consume and then die. Workers obtain their labor income.

Stage 5: (Savings of workers) Workers can save their labor income in bonds or invest in capital. A bond that has claims on consumption units in stage 4 in the next period is traded. Workers use their labor income to purchase the bond or short the bond. The balance of their income is invested in capital for the next period. The net supply of the bond is 0. The economy repeats stages 1 to 5 in the next period.

In this section, we assume that agents have a risk-averse utility function (specifically, Epstein-Zin preferences). An agent's lifelong utility function is

$$U\left(N_t, C_{t+1}\right) = -\psi \frac{N_t^{1+\gamma}}{1+\gamma} + \left(\mathbb{E}C_{t+1}^{\rho}\right)^{\frac{1}{\rho}}.$$

where N_t is his labor supply as a (young) worker in period t, C_{t+1} is his consumption as an (old) capitalist in period $t+1$, and parameter $\rho < 1$ measures risk aversion.

Compared with the baseline model where $K_0 = 1$ is exogenous, we may think that in this section K_0 is endogenized and also a bond market is introduced.

Investors (Capitalists) In this section, for simplicity, we assume that there are no firms. Instead, a capitalist (investor) directly hires workers to produce. This is equivalent to the case where each capitalist owns one firm which hires workers to produce. The value of the firm is the capital income of the firm's owner – the output (revenue) of the firm minus its labor cost. Trading capital is equivalent to trading the value (shares) of a firm in the financial market.

Let us first consider the problem of capitalist j who receives private signals $a_{t+1}+\varepsilon_{jt}$ and $z_t+\delta_{jt}$ (where $\log A_{t+1} \equiv a_{t+1}$); that is, his information set in stage 2 is $\Omega_{jt} = \{K_t, P_t, A^t, a_{t+1}+\varepsilon_{jt}, z_t+\delta_{jt}\}$. He solves

$$V_t(\varepsilon_{jt}, \delta_{jt}, K_{jt}) = \max_{M_{jt}} \left(\mathbb{E}\left[C_{jt}^{\rho}|\Omega_{jt}\right]\right)^{\frac{1}{\rho}} \tag{31}$$

where

$$C_{jt} = P_t M_{jt} + \max_{N_{jt}} \left[A_t(K_{jt} - M_{jt})^{\alpha} N_{jt}^{1-\alpha} - W_t N_{jt}\right]. \tag{32}$$

We explain (32). In stage 2, the capitalist can sell $M_{jt} \leq K_t$ to other capitalists and keep $K_t - M_{jt}$ for production. In stage 3, the capitalist hires labor, N_{jt}, and produces according to production function

$$Y_{jt} = A_t(K_{jt} - M_{jt})^{\alpha} N_{jt}^{1-\alpha}.$$

So the first term of C_{jt} in (32) is the income from selling capital in stage 2 and the second term is the capitalist's production revenue net of labor cost (or his net capital income of production) in stage 3.[20] It is easy to verify later that a capitalist faces no uncertainty about his consumption level C_{jt} given his information set Ω_{jt}, so (31) can be written as

$$V_t(\varepsilon_{jt}, \delta_{jt}, K_{jt}) = \max_{M_{jt}} \mathbb{E}\left[C_{jt}|\Omega_{jt}\right]. \tag{33}$$

We work by backward induction from stage 3 to stage 2. In stage 3, given K_{jt}, M_{jt}, W_t and A_t, capitalist j's first-order condition with respect to N_{jt} in (32) is

$$N_{jt} = \left[\frac{A_t(1-\alpha)}{W_t}\right]^{\frac{1}{\alpha}} (K_{jt} - M_{jt}). \tag{34}$$

From (34), we see that given W_t, a capitalist's labor hiring is *in proportion (linear)* to his capital stock, $K_{jt} - M_{jt}$. By substituting (34) into (32), C_{jt} becomes

$$C_{jt} = P_t M_{jt} + \alpha Y_{jt}, \tag{35}$$

where

$$Y_{jt} = A_t \left[\frac{A_t(1-\alpha)}{W_t}\right]^{\frac{1-\alpha}{\alpha}} (K_{jt} - M_{jt}). \tag{36}$$

The second term of C_{jt} in (35) is capitalist j's production revenue net of labor cost or α proportion of his production revenue Y_{jt}. The production revenue (output) is given by (36), a linear function of his capital holdings, $K_{jt} - M_{jt}$. The linearity of (36) will determine that the production across capitalists can be aggregated and an aggregate production function exists. We then move to stage 2. Let $R_t \equiv \alpha A_t \left[\frac{A_t(1-\alpha)}{W_t}\right]^{\frac{1-\alpha}{\alpha}}$. The optimization problem in stage 2, (33), is transformed to

$$V_t(\varepsilon_{jt}, \delta_{jt}, K_{jt}) = \max_{M_{jt}} \mathbb{E}\left[P_t M_{jt} + R_t(K_{jt} - M_{jt})|\Omega_{jt}\right]. \tag{37}$$

The first-order condition with respect to M_{jt} implies

$$P_t = \mathbb{E}\left[R_t|\Omega_{jt}\right]. \tag{38}$$

It will be shown that the aggregate labor supply N_t and thereby R_t are a function of P_t and thus $P_t = R_t$. So the solution to (37) becomes

$$V_t(\varepsilon_{jt}, \delta_{jt}, K_{jt}) = R_t K_{jt}. \tag{39}$$

[20] Although the selling of capital takes place in stage 2, the receipt of income from this sale will be in stage 4.

Workers Workers are assumed to be homogeneous and have common information set $\Omega_{it} = \{W_t, P_t, A_t, K_t\}$ in stage 3. We analyze the decisions of a representative worker by backward induction from stage 5 to stage 3. In stage 5, after obtaining his labor income, the worker can purchase the bond or invest his income in capital for the next period. Let B_t be the unit of bonds held and K_{t+1} be the capital savings (after trading the bond). Denote the bond price by $\frac{1}{R_{ft}}$, where R_{ft} is the bond yield.[21]

The worker's consumption in period $t+1$ from his bond claim is B_t. The worker will become a capitalist in the next period with private information $a_{t+2} + \varepsilon'$ and $z_{t+1} + \delta'$. His expected consumption from capital savings perceived at the beginning of the next period is thus given by $V_{t+1}(\varepsilon', \delta', K_{t+1}) = R_{t+1}K_{t+1}$ based on (39). Therefore, the worker's total consumption in period $t+1$ is

$$C_{t+1} = B_t + R_{t+1}K_{t+1},$$

with budget constraint

$$B_t \frac{1}{R_{ft}} + K_{t+1} \leq W_t N_t.$$

Denote the ratio of capital savings by $s_t \equiv \frac{K_{t+1}}{W_t N_t}$. The representative worker's problem thus can be written as

$$\max_{N_t} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + N_t W_t \left[\max_{s_t} \left(\mathbb{E}\left[\left(\begin{array}{c} (1-s_t) R_{ft} \\ +s_t R_{t+1} \end{array} \right)^{\rho} | \Omega_{it} \right] \right)^{\frac{1}{\rho}} \right], \tag{40}$$

where the second max is about the bond trade decision in stage 5 and the first max is about the labor supply decision in stage 3. Finally, the zero net supply of the bond means that in equilibrium

$$s_t = 1. \tag{41}$$

The first-order condition with respect to s_t in (40), together with (41), gives

$$\mathbb{E}\left[R_{t+1}|\Omega_{it}\right] - R_{ft} = \frac{-Cov\left(m_{t+1}, R_{t+1}|\Omega_{it}\right)}{\mathbb{E}\left[m_{t+1}|\Omega_{it}\right]} \text{ where } m_{t+1} = \rho C_{t+1}^{\rho-1}$$

or

$$R_{ft} = \frac{\mathbb{E}\left[R_{t+1}^{\rho}|\Omega_{it}\right]}{\mathbb{E}\left[R_{t+1}^{\rho-1}|\Omega_{it}\right]}.$$

Then, the first-order condition with respect to N_t in (40) implies

$$\begin{aligned} \psi N_t^{\gamma} &= W_t \left(\mathbb{E}\left[R_{t+1}^{\rho}|\Omega_{it}\right]\right)^{\frac{1}{\rho}} \\ &= W_t \left(\mathbb{E}\left[R_{t+1}^{\rho}|P_t, A_t, K_t\right]\right)^{\frac{1}{\rho}}. \end{aligned} \tag{42}$$

[21] The bond traded in period t has price $\frac{1}{R_{ft}}$ (units of consumption goods) and is endowed with the claim of 1 unit of consumption goods in the next period $t+1$.

In (42), since W_t is a function of N_t and K_t, the information set $\Omega_{it} = \{W_t, P_t, A_t, K_t\}$ is effectively equivalent to $\{P_t, A_t, K_t\}$.

Now we derive the *aggregate* production function. In equilibrium, we have

$$\int M_{jt} dj = 0.$$

As all the capitalists start with the same level of capital, $K_{jt} = W_{t-1} N_{t-1} = K_t$. By (34), the labor market equilibrium condition thus can be written as

$$N_t = \int N_{jt} dj = \left[\frac{A_t (1-\alpha)}{W_t} \right]^{\frac{1}{\alpha}} K_t. \tag{43}$$

From (36), the *aggregate production* can be written as

$$Y_t = \int Y_{jt} dj = \left[\frac{A_t (1-\alpha)}{W_t} \right]^{\frac{1-\alpha}{\alpha}} A_t K_t. \tag{44}$$

Equations (43) and (44) together imply

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}.$$

So

$$W_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \text{ and } R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha}. \tag{45}$$

Capital evolves as

$$K_{t+1} = (1-\alpha) Y_t = (1-\alpha) A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{46}$$

Applying (45), equation (42) becomes

$$N_t^{(\gamma+1)-(1-\alpha)\alpha} = \frac{\alpha (1-\alpha)^{\alpha}}{\psi} \left[A_t K_t^{\alpha} \right]^{\alpha} \left\{ \mathbb{E} \left[\left(A_{t+1} N_{t+1}^{1-\alpha} \right)^{\rho} | P_t, A_t, K_t \right] \right\}^{\frac{1}{\rho}}. \tag{47}$$

By normalizing $\psi^{-1} \alpha (1-\alpha)^{\alpha} = 1$ and denoting $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$ as in (10), we obtain a key set of equations in equilibrium (we denote $x_t = \log X_t$):

$$\begin{aligned} n_t &= \alpha \theta a_t + \alpha^2 \theta k_t + \frac{\theta}{\rho} \log \mathbb{E} \left[\exp \left(\rho a_{t+1} + \rho (1-\alpha) n_{t+1} \right) | p_t, a_t, k_t \right] && (48) \\ k_{t+1} &= \log(1-\alpha) + a_t + \alpha k_t + (1-\alpha) n_t && (49) \\ p_t &= \log \mathbb{E} \left[\exp \left(\log \alpha + a_t + (1-\alpha)(n_t - k_t) \right) | p_t, a_t, k_t, a_{t+1} + \varepsilon_{jt}, z_t + \delta_{jt} \right], && (50) \end{aligned}$$

where (48) is from (47), (49) is from (46), and (50) is from (38). Here we have used the fact that the relevant economic history up to the beginning of period t can be summarized by a_t and k_t.

In fact, conditions (48) and (50) parallel conditions (10) and (11) in the baseline model, respectively. The only difference in the dynamic model is the additional condition, (49), which gives the law of motion for capital. We conjecture that equilibrium labor takes the form

$$n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + z_t),$$

where n^c, φ, π and ϕ are coefficients to be determined. Proposition 7 summarizes the equilibria.

Proposition 7 *There exists a continuum of equilibria indexed by $0 \leq \sigma_z^2 \leq \frac{\hat{\theta}^2}{4}\sigma_a^2$, in which the price p_t is given by*

$$p_t = \log R_t = [\log \alpha + (1-\alpha) n^c] + [1 + (1-\alpha)\varphi] a_t + (1-\alpha)(\pi - 1) k_t + (1-\alpha)(\phi a_{t+1} + z_t),$$

and

$$n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + z_t)$$

$$k_{t+1} = \log(1-\alpha) + a_t + \alpha k_t + (1-\alpha) n_t$$

$$y_t = a_t + \alpha k_t + (1-\alpha) n_t,$$

where

$$\pi = \frac{\left[\frac{1}{(1-\alpha)^2\theta} - \frac{\alpha}{1-\alpha}\right] - \sqrt{\left[\frac{1}{(1-\alpha)^2\theta} - \frac{\alpha}{1-\alpha}\right]^2 - 4\left(\frac{\alpha}{1-\alpha}\right)^2}}{2}$$

$$\varphi = \frac{\alpha\theta + (1-\alpha)\pi\theta}{1 - (1-\alpha)^2\pi\theta}$$

$$\phi = \frac{\hat{\theta} \pm \sqrt{\hat{\theta}^2 - \frac{4\sigma_z^2}{\sigma_a^2}}}{2} \text{ with } \hat{\theta} \equiv \frac{\theta[1 + (1-\alpha)\varphi]}{1 - \theta(1-\alpha)^2\pi}$$

and

$$n^c = \frac{\theta(1-\alpha)\pi\log(1-\alpha) + \frac{1}{2}\theta\sigma_a^2\Theta + \frac{1}{2}\rho\theta(1-\alpha)^2\sigma_z^2}{1 - \theta(1-\alpha)[1 + \pi(1-\alpha)]}$$

with $\Theta \equiv \rho[(1-\alpha)\phi]^2 - [(1-\alpha)\phi] + \left(1 - \frac{\phi}{\hat{\theta}}\right)\left(\rho[1 + (1-\alpha)\varphi]^2 - [1 + (1-\alpha)\varphi]\right)$.

Proof. *See Appendix.* ■

The intuition behind Proposition 7 is similar to that in our baseline model. While a_{t+1} directly affects the workers' return on savings, its effect on existing capitalists (the old generation) is only through an indirect general equilibrium channel. If high sentiments lead capitalists to speculate that the output and hence the demand for capital will be high in period t, competition will drive up the capital price P_t. After observing a high price P_t, the workers in period t will need to solve a signal extraction problem, leading them to attribute the high price P_t partially to high productivity in the next period. High productivity in the next period increases their incentive to supply labor in the current period. So output and the demand for capital will indeed be high in period t. The initial conjecture of the existing capitalists is thus verified.[22]

Three remarks on the OLG equilibrium are in order. First, if we set $\sigma_z = 0$, then ϕ has two solutions, 0 and $\hat{\theta}$. It is easy to verify that the fully-revealing equilibrium (corresponding to $\sigma_z = 0$ and $\phi = \hat{\theta}$) and the non-revealing equilibrium (corresponding to $\sigma_z = 0$ and $\phi = 0$) are two special cases of the sentiment-driven equilibria in Proposition 7. Sentiment-driven equilibria are more likely when σ_a^2 is higher. Second, the aggregate capital stock, k_t, in our model has a stationary distribution. In fact, based on Proposition 7, we have

$$k_{t+1} = [\log(1-\alpha) + (1-\alpha)\, n^c] + [1 + (1-\alpha)\,\varphi]\, a_t + [\alpha + (1-\alpha)\,\pi]\, k_t + (1-\alpha)\,(\phi a_{t+1} + z_t)\,.$$

We prove that $\pi < 1$ and therefore $0 < \alpha + (1-\alpha)\,\pi < 1$ (see the appendix). Third, the assumption of the utility function with risk neutrality versus risk aversion (i.e., $\rho = 1$ versus $\rho < 1$) affects the equilibrium by altering the constant coefficient n^c only but not the other coefficients (φ, π and ϕ). In fact, ρ appears only in the constant term n^c in Proposition 7. The most important reason for introducing the risk-averse utility is to derive the asset pricing implications, to be analyzed below.

Now we proceed to discuss implications of the OLG model.

Implication 1: Persistence in business cycles Proposition 7 shows that i.i.d. sentiment shocks can generate persistent fluctuations in output and unemployment. Concretely, a sentiment shock z_t in period t influences the labor supply in period t (that is n_t), which in turn affects the capital level in period $t+1$ (that is k_{t+1}). Because the OLG economy is dynamically linked across periods by capital accumulation, z_t can have a persistent effect on asset prices, output and unemployment.

Implication 2: Asset prices over business cycles (cross section) We derive the risk premium in our model, which is defined as $\Delta r_t \equiv \log \mathbb{E}\left[R_{t+1}|\Omega_{it}\right] - \log R_{ft}$. We have the closed-form solution of it.

[22] The equilibrium can be implemented following the approach analyzed in Section 4.1.

Corollary 1 *The risk premium is given by*

$$\begin{aligned}\Delta r_t &= (1-\rho)\,Cov\left(\log\frac{C_{t+1}}{C_t},\log R_{t+1}|\Omega_{it}\right)=(1-\rho)\,Var\left(\log R_{t+1}|\Omega_{it}\right)\\ &= (1-\rho)\left\{\begin{array}{c} Var\left(\left[1+(1-\alpha)\,\varphi\right]a_{t+1}|\phi a_{t+1}+z_t\right)\\ +Var\left[(1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)\right]\end{array}\right\}\\ &= (1-\rho)\left\{\left[1+(1-\alpha)\varphi\right]^2\frac{\sigma_a^2\sigma_z^2}{\phi^2\sigma_a^2+\sigma_z^2}+(1-\alpha)^2\left(\phi^2\sigma_a^2+\sigma_z^2\right)\right\}, \end{aligned} \tag{51}$$

which is increasing in σ_z^2 *when* ϕ *is decreasing in* σ_z^2.

Proof. *See Appendix.* ■

Corollary 1 implies that if ϕ as a function of σ_z^2 takes the form $\phi=\frac{\hat{\theta}+\sqrt{\hat{\theta}^2-\frac{4\sigma_z^2}{\sigma_a^2}}}{2}$ (corresponding to the upper branch of the curve as in Figure 3), the risk premium is increasing in sentiment volatility. Our model hence gives a novel implication of sentiment volatility-driven risk premia, one that depends crucially on the mechanism of the feedback between financial markets and the real economy. Namely, the sentiments in financial markets affect the real economy and thus influence aggregate consumption and asset returns, which in turn impact the risk premium. Corollary 1 gives cross-sectional predictions. The model predicts that an economy with higher investor sentiment volatility in financial markets (i.e., higher σ_z) will have a higher risk premium in asset returns. The intuition behind Corollary 1 is as follows. Return R_{t+1} in the next period $t+1$ depends on a_{t+1} and n_{t+1}. In period t, workers can infer information about a_{t+1} from p_t, a_t, k_t and effectively from $\phi a_{t+1}+z_t$ but has no information about n_{t+1} which is a function of $\phi a_{t+2}+z_{t+1}$. So the conditional variance of R_{t+1} depends on conditional variance $Var\left(\left[1+(1-\alpha)\,\varphi\right]a_{t+1}|\phi a_{t+1}+z_t\right)$ and variance $Var\left[(1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)\right]$. Higher sentiment volatility results in lower informativeness about a_{t+1} and hence higher conditional variance of R_{t+1}, which leads to a bigger gap between the mean of R_{t+1} and its certainty-equivalent value or a higher risk premium.

Implication 3: Asset prices over business cycles (time series) In the OLG model so far (Proposition 7), the sentiment volatility (σ_z) and the risk aversion (ρ) are invariant across time. So the risk premium is constant across time for a given equilibrium path. A direct time-series implication (based on Corollary 1) is, therefore, that if there is an *unexpected* shock to sentiment volatility in the financial market (i.e., an increase in σ_z, for example in a financial crisis), the risk premium increases.

Now we extend the OLG model to allow for regime changes and formally model time-varying risk premia. To highlight the mechanism and for tractability, we consider three cases. First, we consider a deterministic process of time-varying sentiment volatility. Specifically, we assume that sentiment volatility, σ_z, has four realized values with a seasonal cycle of $(\sigma_{z(1)}, \sigma_{z(2)}, \sigma_{z(3)}, \sigma_{z(4)}, \sigma_{z(1)}, ...)$, where $\sigma_{z(1)} > \sigma_{z(2)} = \sigma_{z(3)} = \sigma_{z(4)}$; that is, the sentiment volatility in the first season is higher than those in other three seasons. A possible reason for the seasonal variation in sentiment volatility is seasonal affective disorder (SAD, also known as winter blues or winter depression), which is linked closely with hours of daylight; experimental research in psychology has documented that SAD can generate depression. Kamstra, Kramer and Levi (2003) show clear evidence that the seasonal SAD causes stock market seasonality. Under the above setup, we are able to derive closed-form risk premia.

Corollary 2 *Suppose sentiment volatility has a seasonal cycle of* $(\sigma_{z,t}, \sigma_{z,t+1}, \sigma_{z,t+2}, \sigma_{z,t+3}, \sigma_{z,t+4}, ...) = (\sigma_{z(1)}, \sigma_{z(2)}, \sigma_{z(3)}, \sigma_{z(4)}, \sigma_{z(1)}, ...)$. *The equilibrium of the OLG model exists. The risk premium has a seasonal cycle, which is* $(\Delta r_t, \Delta r_{t+1}, \Delta r_{t+2}, \Delta r_{t+3}, \Delta r_{t+4}, ...) = (\Delta r_{(1)}, \Delta r_{(2)}, \Delta r_{(3)}, \Delta r_{(4)}, \Delta r_{(1)}, ...)$, *where* $\Delta r_{(1)} > \Delta r_{(2)} = \Delta r_{(3)} > \Delta r_{(4)}$.

Proof. *See Appendix.* ■

In a rational expectations framework, Corollary 2 provides a novel perspective to explain seasonal tendencies/calendar effects in financial markets and has implications for return predictability. Because of the feedback between the financial market and the real economy, the season with higher sentiment volatility also has a higher risk premium. From (51), we can see that the risk premium in the current period depends on not only the equilibrium in the current period but also the equilibrium in the next period. Solving the risk premia under a seasonal cycle of sentiment volatility is not trivial. Figure 4 illustrates an example of time-series risk premia, where the parameter values are $\alpha = 0.3$, $\gamma = 1$, $\rho = -3$, $\sigma_a = 0.1$, $\sigma_{z(1)} = 0.02$, $\sigma_{z(2)} = \sigma_{z(3)} = \sigma_{z(4)} = 0.01$.[23]

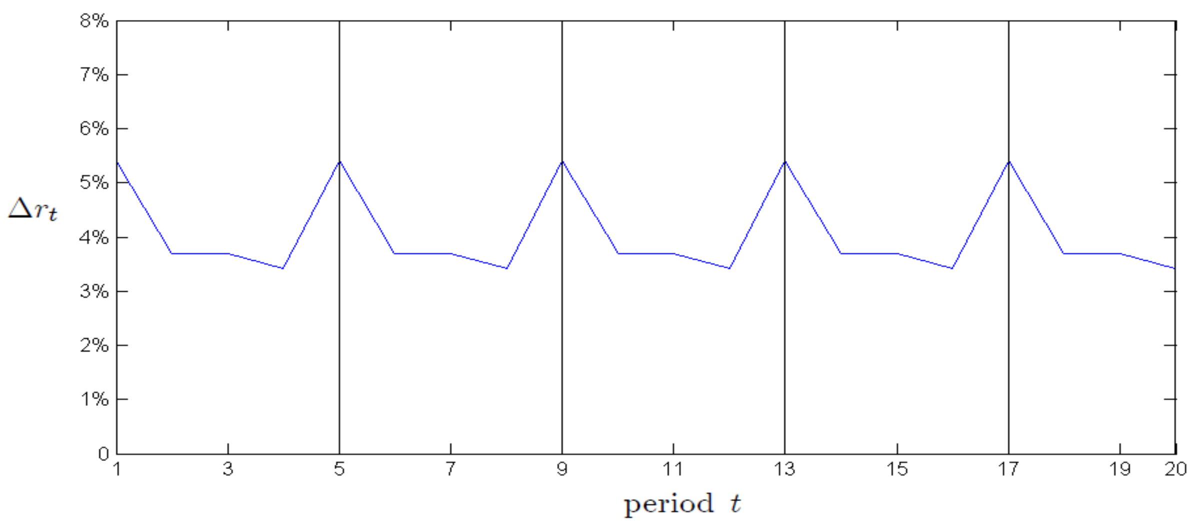

Figure 4: Time-series risk premia (annualized) under a seasonal cycle of sentiment volatility σ_z

[23] Parameter choice $\sigma_a = 0.1$ is to capture the large fluctuation in output in the seasonal frequency (see, e.g., Wen (2002)).

Second, consider the case where there is a stochastic switch between the fully-revealing equilibrium and the non-revealing equilibrium across time. Specifically, we assume a Markov process of regime changes with stochastic matrix

$$\begin{pmatrix} q_{F,F} & 1-q_{F,F} \\ q_{N,F} & 1-q_{N,F} \end{pmatrix} \tag{52}$$

where $q_{F,N}$ is the transition probability from the fully-revealing equilibrium to the full-revealing equilibrium and $q_{N,F}$ is the transition probability from the non-revealing equilibrium to the fully-revealing equilibrium. We have the following corollary.

Corollary 3 *Suppose the fully-revealing equilibrium and the non-revealing equilibrium follow the Markov process of (52). The equilibrium of the OLG model exists. The risk premium, Δr_t, follows the Markov process, in which Δr_t is higher in the state of the non-revealing equilibrium and lower in the state of the fully-revealing equilibrium (under a sufficient condition that $|q_{F,N} - q_{N,F}|$ is not too high).*

Proof. *See Appendix.* ■

In the dynamic rational-expectations equilibrium, the equilibrium in period t needs to incorporate the expectation of regime changes in the future. So the equilibrium is unlike the case with an unexpected shock to regime changes. Solving the equilibrium is not trivial and the details are provided in the appendix. Corollary 3 formalizes stochastic time-varying risk premia. Again, our explanation of time-varying risk premia is built into a rational-expectation framework and crucially depends on the mechanism of the feedback between the financial market and the real economy.

Third, we consider regime changes of time-varying risk aversion (and constant sentiment volatility). Specifically, we assume that the risk aversion ρ has two states (ρ_L, ρ_H), where $\rho_L < \rho_H$, and follows a Markov process with stochastic matrix

$$\begin{pmatrix} \xi_{H,H} & 1-\xi_{H,H} \\ \xi_{L,H} & 1-\xi_{L,H} \end{pmatrix} \tag{53}$$

where $\xi_{H,H}$ is the transition probability from risk aversion ρ_H to risk aversion ρ_H and $\xi_{L,H}$ is the transition probability from risk aversion ρ_L to risk aversion ρ_H. Workers when born know their own risk aversion but when making their labor decisions need to form expectations about the risk aversion of the next generation of workers. We have the following corollary.

Corollary 4 *Suppose the risk aversion ρ follows the Markov process of (53). The equilibrium of the OLG model exists. The risk premium, Δr_t, follows the Markov process, in which Δr_t is higher in the regime of $\rho = \rho_L$ and is lower in the regime of $\rho = \rho_H$ (under a sufficient condition that*

$\left|\xi_{H,H} - \xi_{L,H}\right|$ *is not too high). In particular, the variation in the risk premium across the two regimes is increasing in sentiment volatility* (σ_z).

Proof. *See Appendix.* ■

In Corollary 4, it may not be surprising that the risk premium is higher when risk aversion is higher. What we want to emphasize, however, is the amplification effect of sentiment volatility. Only in the presence of sentiment and the feedback effect, does the time-varying risk aversion play a large role in generating time-varying risk premia. In fact, we prove that given ρ_L and ρ_H the variation (difference) in the risk premium between the two regimes is increasing in sentiment volatility σ_z.

Finally, we note that due to the computational constraints, we cannot obtain analytical solutions of non-linear asymmetric equilibria for the OLG model in general cases. For some special cases however we can show that the risk premium is asymmetric.[24]

7 Conclusion

In this paper, we study how the financial sector can affect the aggregate real economy through the information channel. In the rational expectations framework, we show that investors' sentiments affect financial market prices which in turn influence real activities. Because of the two-way feedback between the financial sector and the real sector, a small sentiment shock in the financial market can be amplified and can have a large impact on the real economy. The sentiment-driven equilibria also have implications for non-linear asset prices, discontinuity in asset prices, and cross-country comovements in asset prices and real output. Under informational frictions, investors' perception of synchronization across economies can lead to actual synchronization. In a dynamic economy, sentiment-driven fluctuations can also generate persistence in business cycles and have cross-sectional and time-series implications for asset prices over business cycles. The main purpose of our paper is to illustrate the possibility of sentiment-driven fluctuations in asset prices and real output. Embedding our mechanism in a full-fledged DSGE model and quantifying the role of sentiments is left for future research.

[24] Specifically, as in (24), consider the case where the equilibrium is fully-revealing when $a_{t+1} \geq 0$ and it is non-revealing when $a_{t+1} < 0$. In this case, the equilibrium in Proposition 7 becomes

$$n_t = n^c + \varphi a_t + \pi k_t + \phi \cdot \mathbf{1}_S (a_{t+1}) \cdot a_{t+1} + \phi^c \cdot \mathbf{1}_{S^c} (a_{t+1}),$$

$$p_t = \log R_t = [\log \alpha + (1-\alpha) n^c] + [1 + (1-\alpha)\varphi] a_t + (1-\alpha)(\pi - 1) k_t + (1-\alpha) [\phi \cdot \mathbf{1}_S (a_{t+1}) \cdot a_{t+1} + \phi^c \cdot \mathbf{1}_{S^c} (a_{t+1})],$$

with the expressions of k_{t+1} and y_t unchanged, where $S = \{a_{t+1} \geq 0\}$ and S^c is the complement set of S, and n^c, φ, π, ϕ and ϕ^c are coefficients. Then, based on the result of Corollary 3, the risk premium Δr_t is asymmetric for $a_{t+1} \geq 0$ and $a_{t+1} < 0$.

Appendix

A Proofs

Proof of Proposition 3: We prove that P_0 and N_1 defined in equations (14) and (15) constitute a rational expectations equilibrium. Under (14), the first condition, (10), becomes

$$\begin{aligned}\log N_1 &= \theta \log \left\{\mathbb{E}\left[A_2|\phi a_2 + z\right]\right\} \\ &= \theta \left\{\mathbb{E}\left[a_2|\phi a_2 + z\right] + \frac{1}{2} var(a_2|\phi a_2 + z)\right\},\end{aligned}$$

where

$$\mathbb{E}\left[a_2|\phi a_2 + z\right] = -\frac{1}{2}\sigma_a^2 + \frac{\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}\left(\phi a_2 + z + \frac{1}{2}\phi\sigma_a^2\right)$$

and

$$var(a_2|\phi a_2 + z) = \sigma_a^2 - \frac{\phi^2\sigma_a^4}{\phi^2\sigma_a^2 + \sigma_z^2}.$$

Comparing terms with the conjecture $\log N_1 = \bar{n} + \phi a_2 + z$ yields

$$\phi a_2 + z = \frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}\left(\phi a_2 + z\right)$$

or

$$1 = \frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}, \tag{A.1}$$

and

$$\bar{n} = \theta\left\{-\frac{1}{2}\sigma_a^2 + \frac{\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}\frac{1}{2}\phi\sigma_a^2 + \frac{1}{2}\sigma_a^2 - \frac{1}{2}\frac{\phi^2\sigma_a^4}{\phi^2\sigma_a^2 + \sigma_z^2}\right\} = 0.$$

Solving (A.1) with respect to ϕ gives

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha},$$

where $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4}\sigma_a^2$.

Under (15), equation (11) becomes

$$\log P_0 = \log\alpha + (1-\alpha)\log N_1 = \log\alpha + (1-\alpha)\left(\phi a_2 + z\right),$$

that is, $\bar{p} = 0$.

Proof of Proposition 4: First, we conjecture $p_0 = u^* + \log\alpha + (1-\alpha)\left(\phi a_2 + z\right)$, where u^* is a constant to be determined. After observing this price in period 1, workers' information extraction problem is the same as that in Proposition 3, so their labor supply is still $\log N_1 = \phi a_2 + z$. Given this labor supply, the dividend is still $\log R_1 = \log \alpha + (1-\alpha)(\phi a_2 + z)$. Hence, we have $P_0 = e^{u^*} R_1$.

Second, conditional on a realization of P_0, it is optimal for investor j to buy capital if and only if

$$\mathbb{E}\left(R_1 e^{u_j} | p_0, s_{j0}, l_{j0}\right) \geq P_0$$

or $\mathbb{E}\left(e^{u_j} | p_0, s_{j0}, l_{j0}\right) \geq e^{u^*}$. We express u_j in terms of p_0, s_{j0} and l_{j0}. It is easy to obtain

$$s_{j0} - \frac{\frac{p_0 - (u^* + \log \alpha)}{1-\alpha}}{\phi} + \frac{l_{j0}}{\phi} = u_j + \left(\frac{1}{\phi}\delta_j + \varepsilon_j\right),$$

that is, the combination, $s_{j0} - \frac{\frac{p_0-(u^*+\log\alpha)}{1-\alpha}}{\phi} + \frac{l_{j0}}{\phi}$, provides a signal about u_j. Thus, condition $\mathbb{E}\left(e^{u_j}|p_0, s_{j0}, l_{j0}\right) \geq e^{u^*}$ can be transformed to $\phi s_j + l_{j0} \geq \frac{p_0-(u^*+\log\alpha)}{1-\alpha} + \phi \frac{\sigma_u^2 + \frac{1}{\phi^2}\sigma_\delta^2 + \sigma_\varepsilon^2}{\sigma_u^2} u^*$. This is (17).

Third, given the demand schedule (17), market clearing implies

$$\frac{pr\left(\phi s_{j0} + l_{j0} < \frac{p_0-(u^*+\log\alpha)}{1-\alpha} + \phi \frac{\sigma_u^2 + \frac{1}{\phi^2}\sigma_\delta^2 + \sigma_\varepsilon^2}{\sigma_u^2} u^*\right)}{pr\left(\phi s_{j0} + l_{j0} \geq \frac{p_0-(u^*+\log\alpha)}{1-\alpha} + \phi \frac{\sigma_u^2 + \frac{1}{\phi^2}\sigma_\delta^2 + \sigma_\varepsilon^2}{\sigma_u^2} u^*\right)} = \bar{d}.$$

Note that $s_{j0} + \frac{l_{j0}}{\phi} \sim N\left(a_2 + \frac{z}{\phi}, \sigma_u^2 + \sigma_\varepsilon^2 + \frac{\sigma_\delta^2}{\phi^2}\right)$. Because the market clearing equation is true for any p_0, we obtain $u^* = \frac{\Phi^{-1}\left(\frac{\bar{d}}{1+\bar{d}}\right)\sigma_u^2}{\sqrt{\sigma_u^2 + \sigma_\varepsilon^2 + \frac{\sigma_\delta^2}{\phi^2}}}$.

Proof of Proposition 5: We show that the combination of (20), (23) and (22) satisfies conditions (18) and (19). Note that W_1 is a function of N_1, so given (22) we must have (21). First, because W_1 is in the form of (21), a worker can infer a_2^H perfectly by comparing W_1 with P_0. The *effective* information set of the workers becomes the same across workers, namely $\Omega_{i1} = \Omega_1 = \{R_1, W_1, P_0, a_2^H\}$. So workers can make an identical decision in period 1; that is, symmetric equilibrium among workers, condition (18), still applies.

Condition (18) becomes

$$\begin{aligned}
\log N_1 &= \theta \log\left\{\mathbb{E}\left[A_2 | \phi a_2^I + z, a_2^H\right]\right\} \\
&= \theta\left\{\mathbb{E}\left[a_2^I + a_2^H | \phi a_2^I + z, a_2^H\right] + \frac{1}{2} var(a_2^I + a_2^H | \phi a_2^I + z, a_2^H)\right\} \\
&= \theta\left\{\mathbb{E}\left[a_2^I | \phi a_2^I + z\right] + \frac{1}{2} var(a_2^I | \phi a_2^I + z)\right\} + \theta a_2^H. \qquad \text{(A.2)}
\end{aligned}$$

We have

$$\mathbb{E}\left[a_2^I|\phi a_2^I+z\right]=-\frac{1}{2}\sigma_I^2+\frac{\phi\sigma_I^2}{\phi^2\sigma_I^2+\sigma_z^2}\left(\phi a_2^I+z+\frac{1}{2}\phi\sigma_I^2\right)$$

and

$$var(a_2^I|\phi a_2^I+z)=\sigma_I^2-\frac{\left(\phi\sigma_I^2\right)^2}{\phi^2\sigma_I^2+\sigma_z^2}.$$

Under condition (23), it is easy to verify that (A.2) becomes (22).

Next, we turn to the equilibrium condition of (19), which by substituting (22) becomes

$$\begin{aligned}\log P_0 &= \log\alpha+\log\mathbb{E}\left[\exp\left[(1-\alpha)\log N_1\right]|\phi a_2^I+z,a_2^I+\varepsilon_j\right]\\ &= \log\alpha+(1-\alpha)\left(\phi a_2^I+z\right).\end{aligned}$$

This is (20), so condition (19) is verified.

Proof of Proposition 6: By (27), $p(a,z)<0$. Hence the workers can perfectly infer a_2 if $\log P_0\geq\log\alpha$. When the price falls below $\log\alpha$, the workers know that $a_2<0$ and learn about a_2 from price $p(a_2,z)$ and effectively from $\exp(a_2)+\exp(z)$. Conditional on $a_2<0$, a_2 and z are independent and identically distributed. So, by symmetry we have $\mathbb{E}\left[\exp(a_2)|\exp(a_2)+\exp(z)\right]=\mathbb{E}\left[\exp(z)|\exp(a_2)+\exp(z)\right]$. Since $\mathbb{E}\left[\exp(a_2)+\exp(z)|\exp(a_2)+\exp(z)\right]=\exp(a_2)+\exp(z)$, it then follows that $\mathbb{E}\left[\exp(a_2)|\exp(a_2)+\exp(z)\right]=\frac{\exp(a_2)+\exp(z)}{2}$. Equation (28) is then obtained by (26).

Proof of Proposition 7: Given the price p_t, the workers' effective information set is $\{a_t,k_t,\phi a_{t+1}+z_t\}$. Equation (48) becomes

$$n_t=\alpha\theta a_t+\alpha^2\theta k_t+\frac{\theta}{\rho}\log\mathbb{E}\left[\exp\left(\rho a_{t+1}+\rho\left(1-\alpha\right)n_{t+1}\right)|k_t,a_t,\phi a_{t+1}+z_t\right].$$

Substituting $n_{t+1}=n^c+\varphi a_{t+1}+\pi k_{t+1}+(\phi a_{t+2}+z_{t+1})$ and using $k_{t+1}=\log(1-\alpha)+a_t+\alpha k_t+(1-\alpha)\,n_t$, we obtain

$$n_t=\left\{\begin{array}{l}\theta(1-\alpha)\left[n^c+\pi\log(1-\alpha)\right]\\ +\left[\alpha\theta+\theta(1-\alpha)\pi\right]a_t+\left[\alpha^2\theta+\theta(1-\alpha)\pi\alpha\right]k_t+\theta(1-\alpha)^2\pi n_t\\ +\frac{\theta}{\rho}\log\mathbb{E}\left[\exp\left[\rho(1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)\right]|\phi a_{t+1}+z_t\right]\\ +\frac{\theta}{\rho}\mathbb{E}\left[\rho\left[1+(1-\alpha)\varphi\right]a_{t+1}|\phi a_{t+1}+z_t\right]\\ +\frac{1}{2}\frac{\theta}{\rho}var\left[\rho\left[1+(1-\alpha)\varphi\right]a_{t+1}|\phi a_{t+1}+z_t\right]\end{array}\right\}. \qquad (A.3)$$

Comparing terms of (A.3) with $n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + z_t)$ yields

$$\pi = \frac{\alpha^2\theta + \theta(1-\alpha)\alpha\pi}{1-\theta(1-\alpha)^2\pi}$$

or $\pi = \theta\alpha^2 + (1-\alpha)\pi\theta\left[\alpha + (1-\alpha)\pi\right]$. We assume that the economy is stable, which requires $(1-\alpha)\,\theta\left[\alpha + (1-\alpha)\pi\right] < 1$. Because $\theta = \frac{1}{\gamma+1-\alpha(1-\alpha)}$, it is true that $\theta < \frac{1}{1-\alpha}$ or $\theta\,(1-\alpha) < 1$. When $\pi < 1$, we have $\alpha + (1-\alpha)\pi < 1$. Hence, $(1-\alpha)\,\theta\left[\alpha + (1-\alpha)\pi\right] < 1$. We focus on the case of $\pi < 1$. We solve the quadratic equation with respect to π:

$$G(\pi) \equiv \theta\alpha^2 + (1-\alpha)\pi\theta\left[\alpha + (1-\alpha)\pi\right] - \pi = 0. \tag{A.4}$$

Notice that $G(\pi = 0) = \theta\alpha^2 > 0$ and $G(\pi = 1) = \theta\alpha^2 + (1-\alpha)\theta - 1 < 0$; the latter is true because $\theta = \frac{1}{\gamma+1-\alpha(1-\alpha)}$ and $\frac{1}{\gamma+1-\alpha(1-\alpha)} < \frac{1}{1-\alpha+\alpha^2}$ under $\gamma > 0$ and thus $\theta < \frac{1}{1-\alpha+\alpha^2}$. Therefore, by the intermediate value theorem, there is a unique solution of π that satisfies $0 < \pi < 1$, which is

$$\pi = \frac{\left[\frac{1}{(1-\alpha)^2\theta} - \frac{\alpha}{1-\alpha}\right] - \sqrt{\left[\frac{1}{(1-\alpha)^2\theta} - \frac{\alpha}{1-\alpha}\right]^2 - 4\left(\frac{\alpha}{1-\alpha}\right)^2}}{2}.$$

And φ is given by

$$\varphi = \frac{\alpha\theta + (1-\alpha)\pi\theta}{1-(1-\alpha)^2\pi\theta}.$$

As for ϕ, we have

$$1 = \theta\frac{\left[1 + (1-\alpha)\varphi\right]}{1-\theta(1-\alpha)^2\pi}\frac{\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}. \tag{A.5}$$

We define $\hat{\theta} = \frac{\theta[1+(1-\alpha)\varphi]}{1-\theta(1-\alpha)^2\pi}$. Thus,

$$1 = \hat{\theta}\frac{\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2},$$

that is, $\phi = \frac{\hat{\theta}\pm\sqrt{\hat{\theta}^2 - \frac{4\sigma_z^2}{\sigma_a^2}}}{2}$. Once we have ϕ, we can solve for the constant coefficient n^c:

$$n^c = \frac{\theta(1-\alpha)\pi\log(1-\alpha) + \frac{1}{2}\theta\sigma_a^2\begin{bmatrix}\begin{pmatrix}\rho\left[(1-\alpha)\phi\right]^2 \\ -\left[(1-\alpha)\phi\right]\end{pmatrix} \\ +\left(1-\frac{\phi}{\hat{\theta}}\right)\begin{pmatrix}\rho\left[1+(1-\alpha)\varphi\right]^2 \\ -\left[1+(1-\alpha)\varphi\right]\end{pmatrix}\end{bmatrix} + \frac{1}{2}\rho\theta(1-\alpha)^2\sigma_z^2}{1-\theta(1-\alpha)\left[1+\pi(1-\alpha)\right]}.$$

Proof of Corollary 1: We first calculate R_{ft}, which is given by $R_{ft} = \frac{\mathbb{E}\left[R_{t+1}^{\rho}|k_t,a_t,\phi a_{t+1}+z_t\right]}{\mathbb{E}\left[R_{t+1}^{\rho-1}|k_t,a_t,\phi a_{t+1}+z_t\right]}$. Using the expression of R_t in Proposition 7, we obtain

$$\begin{aligned} r_{ft} &= \log R_{ft} \\ &= \left\{ \begin{array}{l} [\log\alpha + (1-\alpha)\, n^c] + (1-\alpha)\,(\pi-1)\, k_{t+1} \\ -\frac{1}{2}\,[1+(1-\alpha)\varphi]\,\sigma_a^2 + \frac{[1+(1-\alpha)\varphi]\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2}\left(\phi a_{t+1} + \sigma_z z_t + \frac{1}{2}\phi\sigma_a^2\right) \\ +\frac{1}{2}\left[[1+(1-\alpha)\varphi]^2\,\sigma_a^2 - \frac{\left([1+(1-\alpha)\varphi]\phi\sigma_a^2\right)^2}{\phi^2\sigma_a^2+\sigma_z^2}\right](2\rho-1) \\ +\frac{1}{2}\sigma_a^2\left[-[(1-\alpha)\phi] + (2\rho-1)\,[(1-\alpha)\phi]^2\right] + \frac{1}{2}\,(2\rho-1)\,(1-\alpha)^2\sigma_z^2 \end{array} \right\}. \end{aligned}$$

Next, we can work out the expectation of R_{t+1} in log, which is

$$\begin{aligned} & \log\mathbb{E}\left(R_{t+1}|k_t,a_t,\phi a_{t+1}+\sigma_z z_t\right) \\ = & \left\{ \begin{array}{l} [\log\alpha + (1-\alpha)\, n^c] + (1-\alpha)\,(\pi-1)\, k_{t+1} \\ -\frac{1}{2}\,[1+(1-\alpha)\varphi]\,\sigma_a^2 + \frac{[1+(1-\alpha)\varphi]\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2}\left(\phi a_{t+1} + \sigma_z z_t + \frac{1}{2}\phi\sigma_a^2\right) \\ +\frac{1}{2}\left[[1+(1-\alpha)\varphi]^2\,\sigma_a^2 - \frac{\left([1+(1-\alpha)\varphi]\phi\sigma_a^2\right)^2}{\phi^2\sigma_a^2+\sigma_z^2}\right] \\ +\frac{1}{2}\sigma_a^2\left(-[(1-\alpha)\phi] + [(1-\alpha)\phi]^2\right) + \frac{1}{2}(1-\alpha)^2\sigma_z^2 \end{array} \right\}. \end{aligned}$$

Therefore, the risk premium Δr_t is

$$\log\mathbb{E}\left[R_{t+1}|k_t,a_t,\phi a_{t+1}+z_t\right] - \log R_{ft} = (1-\rho)\left\{ \begin{array}{l} Var\left([1+(1-\alpha)\,\varphi]\, a_{t+1}|\phi a_{t+1}+z_t\right) \\ +Var\left[(1-\alpha)\,(\phi a_{t+2}+z_{t+1})\right] \end{array} \right\}$$

$$= (1-\rho)\left\{\left[[1+(1-\alpha)\varphi]^2\,\sigma_a^2 - \frac{\left([1+(1-\alpha)\varphi]\,\phi\sigma_a^2\right)^2}{\phi^2\sigma_a^2+\sigma_z^2}\right] + (1-\alpha)^2\left(\phi^2\sigma_a^2+\sigma_z^2\right)\right\} \tag{A.6}$$

$$= (1-\rho)\left\{[1+(1-\alpha)\varphi]^2\,\frac{\sigma_a^2\sigma_z^2}{\phi^2\sigma_a^2+\sigma_z^2} + (1-\alpha)^2\left(\phi^2\sigma_a^2+\sigma_z^2\right)\right\}.$$

It is easy to confirm that $\Delta r_t = (1-\rho)\, Cov\left(\log\frac{C_{t+1}}{C_t}, \log R_{t+1}|\Omega_{it}\right) = (1-\rho)\, Var\left(\log R_{t+1}|\Omega_{it}\right)$ by noting $\frac{C_{t+1}}{C_t} = \frac{1-\alpha}{\alpha}R_{t+1}$.

Finally, we prove that the term in (A.6) is decreasing in ϕ. Considering that $\phi^2\sigma_a^2 + \sigma_z^2 = \theta\frac{[1+(1-\alpha)\varphi]}{1-\theta(1-\alpha)^2\pi}\phi\sigma_a^2$ by (A.5), we have

$$\begin{aligned} & -\frac{\left([1+(1-\alpha)\varphi]\,\phi\sigma_a^2\right)^2}{\phi^2\sigma_a^2+\sigma_z^2} + (1-\alpha)^2\left(\phi^2\sigma_a^2+\sigma_z^2\right) \\ = & \ [1+(1-\alpha)\varphi]\left(\phi\sigma_a^2\right)\left\{\frac{(1-\alpha)^2\theta}{1-\theta(1-\alpha)^2\pi} - \frac{1-\theta(1-\alpha)^2\pi}{\theta}\right\}. \end{aligned} \tag{A.7}$$

We prove the last term in (A.7) is negative. Note that

$$\begin{aligned} & \frac{(1-\alpha)^2\theta}{1-\theta(1-\alpha)^2\pi} - \frac{1-\theta(1-\alpha)^2\pi}{\theta} < 0 \\ \Leftrightarrow \quad & \theta\left[(1-\alpha)+(1-\alpha)^2\pi\right] < 1 \\ \Leftrightarrow \quad & \pi < \frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2}, \end{aligned}$$

where the last step is obtained by substituting $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$. The relation that $\pi < \frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2}$ is true if and only if $G(\pi = \frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2}) < 0$ is true, where function $G(\pi)$ is defined in (A.4). We verify that $G(\pi = \frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2}) < 0$ is true. Concretely,

$$\begin{aligned} & \theta\alpha^2 + (1-\alpha)\theta\frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2}\left[\alpha+(1-\alpha)\frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2}\right] - \frac{\gamma+\alpha^2}{1-2\alpha+\alpha^2} < 0 \\ \Leftrightarrow \quad & \frac{\alpha^2}{\gamma+1-(1-\alpha)\alpha} + \frac{\gamma+\alpha^2}{(1-\alpha)^2}\left[\frac{\alpha+\gamma}{\gamma+1-(1-\alpha)\alpha} - 1\right] < 0 \\ \Leftrightarrow \quad & -\gamma < 0, \end{aligned}$$

where the second line is obtained by substituting $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$.

Proof of Corollary 2: First, we solve the OLG equilibrium. We conjecture that the labor decision rule in season κ is $n_{t,\kappa} = n^{c,k} + \varphi_\kappa a_t + \pi_\kappa k_t + (\phi_\kappa a_{t+1} + z_t)$, where $\kappa = 1, 2, 3$ and 4, and $Var\left(z_t\right) = \sigma^2_{z,t} = \sigma^2_{z(\kappa)}$. Based on (48), we have

$$n_{t,\kappa} = \left\{ \begin{array}{l} \alpha\theta a_t + \alpha^2\theta k_t + (1-\alpha)\,\theta n^{c,(k\ \%\ 4)+1} \\ +\frac{\theta}{\rho}\log\mathbb{E}\left\{\exp\left[\begin{array}{l} \rho a_{t+1} + \rho(1-\alpha) \\ \cdot\left(\begin{array}{l} \varphi_{(k\ \%\ 4)+1}a_{t+1} + \left(\phi_{(k\ \%\ 4)+1}a_{t+2} + z_{t+1}\right) \\ +\pi_{(k\ \%\ 4)+1}\left[\log(1-\alpha) + a_t + \alpha k_t + (1-\alpha)\,n_{t,\kappa}\right] \end{array}\right) \end{array}\right] |\phi_\kappa a_{t+1} + z_t, a_t\right\} \end{array} \right\}.$$

Comparing terms with $n_{t,\kappa} = n^{c,\kappa} + \varphi_\kappa a_t + \pi_\kappa k_t + (\phi_\kappa a_{t+1} + z_t)$, we obtain $\pi_\kappa = \pi$ and $\varphi_\kappa = \varphi$, where π and φ are as in Proposition 7. The coefficient ϕ_κ solves $1 = \hat{\theta}\frac{\phi_\kappa\sigma_a^2}{\phi_\kappa^2\sigma_a^2+\sigma^2_{z(\kappa)}}$; we focus on the upper branch solution of ϕ_κ as shown in Figure 3, so ϕ_κ is decreasing in $\sigma_{z(\kappa)}$. Since $\sigma_{z(1)} > \sigma_{z(2)} = \sigma_{z(3)} = \sigma_{z(4)}$, we have that $\phi_1 < \phi_2 = \phi_3 = \phi_4$.

Next, we calculate the risk premium. It is easy to find the risk premium in season κ is

$$\begin{aligned} \Delta r_{(\kappa)} = (1-\rho) & \left\{ \begin{array}{l} Var\left(\left[1+(1-\alpha)\,\varphi\right]a_{t+1}|\phi_\kappa a_{t+1} + z_t\right) \\ +Var\left[(1-\alpha)\left(\phi_{(k\ \%\ 4)+1}a_{t+2} + z_{t+1}\right)\right] \end{array} \right\} \\ = \quad (1-\rho) & \left\{\left[1+(1-\alpha)\varphi\right]^2\sigma_a^2 - \frac{\left(\left[1+(1-\alpha)\varphi\right]\phi_\kappa\sigma_a^2\right)^2}{\phi_\kappa^2\sigma_a^2 + \sigma^2_{z(\kappa)}} + (1-\alpha)^2\left(\phi^2_{(k\ \%\ 4)+1}\sigma_a^2 + \sigma^2_{z((k\ \%\ 4)+1)}\right)\right\}. \end{aligned} \tag{A.8}$$

Now we prove that $\Delta r_{(1)} > \Delta r_{(2)} = \Delta r_{(3)} > \Delta r_{(4)}$. Considering that $\phi_\kappa^2\sigma_a^2+\sigma_{z(\kappa)}^2 = \theta\frac{[1+(1-\alpha)\varphi]}{1-\theta(1-\alpha)^2\pi}\phi_\kappa\sigma_a^2$, similar to (A.7), we can transform the last two terms in (A.8) to

$$
\begin{aligned}
&-\frac{\left([1+(1-\alpha)\varphi]\,\phi_\kappa\sigma_a^2\right)^2}{\phi_\kappa^2\sigma_a^2+\sigma_{z(\kappa)}^2} + (1-\alpha)^2\left(\phi_{(k\ \%\ 4)+1}^2\sigma_a^2+\sigma_{z((k\ \%\ 4)+1)}^2\right) \\
= &\left\{\begin{array}{c} [1+(1-\alpha)\varphi]\left(\phi_\kappa\sigma_a^2\right) \\ \cdot\left[\frac{(1-\alpha)^2\theta}{1-\theta(1-\alpha)^2\pi}-\frac{1-\theta(1-\alpha)^2\pi}{\theta}\right] \end{array}\right\} + (1-\alpha)^2\left[\theta\frac{[1+(1-\alpha)\varphi]}{1-\theta(1-\alpha)^2\pi}\sigma_a^2\right]\left(\phi_{(k\ \%\ 4)+1}-\phi_\kappa\right),
\end{aligned}
$$

where we have proved that the term $\frac{(1-\alpha)^2\theta}{1-\theta(1-\alpha)^2\pi}-\frac{1-\theta(1-\alpha)^2\pi}{\theta}$ is negative in (A.7). Therefore, by $\phi_1 < \phi_2 = \phi_3 = \phi_4$, we have $\Delta r_{(1)} > \Delta r_{(2)} = \Delta r_{(3)} > \Delta r_{(4)}$.

Proof of Corollary 3: First, we solve the OLG equilibrium under regime changes. We conjecture that the labor decision rule is $n_{t,F} = n^{c,F} + \varphi a_t + \pi k_t + \phi a_{t+1}$ in the state of the perfect-revealing equilibrium and is $n_{t,N} = n^{c,N} + \varphi a_t + \pi k_t$ in the state of the non-revealing equilibrium. Let $n_{t,F} \equiv \log N_{t,F}$ and $n_{t,N} \equiv \log N_{t,N}$.

Suppose the current state in period t is the fully-revealing equilibrium. Based on (47), we have

$$
N_{t,F}^{(\gamma+1)-(1-\alpha)\alpha} = \frac{\alpha(1-\alpha)^\alpha}{\psi}\left[A_tK_t^\alpha\right]^\alpha\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left[\left(A_{t+1}N_{t+1,F}^{1-\alpha}\right)^\rho|P_t,A_t,K_t\right] \\ +(1-q_{F,F})\,\mathbb{E}\left[\left(A_{t+1}N_{t+1,N}^{1-\alpha}\right)^\rho|P_t,A_t,K_t\right] \end{array}\right\}. \quad (A.9)
$$

By normalizing $\psi^{-1}\alpha(1-\alpha)^\alpha = 1$ and denoting $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$ as in (10), (A.9) can be transformed to

$$
n_{t,F} = \left[\begin{array}{l} [\alpha\theta+\theta(1-\alpha)\pi]\,a_t \\ +\left[\alpha^2\theta+\theta(1-\alpha)\pi\alpha\right]k_t \\ +\theta\left[1+(1-\alpha)\varphi\right]a_{t+1} \\ +\theta(1-\alpha)^2\pi n_{t,H} \end{array}\right] + \left\{\begin{array}{l} \frac{\theta}{\rho}\log\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left[\exp\left[\begin{array}{c}\rho(1-\alpha)\,n^{c,F} \\ +(1-\alpha)\rho\phi a_{t+2}\end{array}\right]|a_{t+1},a_t\right] \\ +(1-q_{F,F})\,\mathbb{E}\left[\exp\left[\rho(1-\alpha)\,n^{c,N}\right]|a_{t+1},a_t\right] \end{array}\right\} \\ +\theta(1-\alpha)\pi\log(1-\alpha) \end{array}\right\}.
$$

By comparing terms with $n_{t,F} = n^{c,F} + \varphi a_t + \pi k_t + \phi a_{t+1}$, we obtain π and φ as in Proposition 7 and $\phi = \hat{\theta}$.

Similarly, suppose the current state in period t is the non-revealing equilibrium. We obtain

$$
n_{t,N} = \left\{\begin{array}{l} \left\{[\alpha\theta+\theta(1-\alpha)\pi]\,a_t+\left[\alpha^2\theta+\theta(1-\alpha)\pi\alpha\right]k_t+\theta(1-\alpha)^2\pi n_{t,N}\right\} \\ +\frac{\theta}{\rho}\log\left\{\begin{array}{l} q_{N,F}\mathbb{E}\left[\exp\left[\rho(1-\alpha)\,n^{c,F}+\rho\left[1+(1-\alpha)\varphi\right]a_{t+1}+(1-\alpha)\rho\phi a_{t+2}\right]|a_t\right] \\ +(1-q_{N,F})\,\mathbb{E}\left[\exp\left[\rho(1-\alpha)\,n^{c,N}+\rho\left[1+(1-\alpha)\varphi\right]a_{t+1}\right]|a_t\right] \end{array}\right\} \\ +\theta(1-\alpha)\pi\log(1-\alpha) \end{array}\right\}.
$$

Comparing terms with $n_{t,N} = n^{c,N} + \varphi a_t + \pi k_t$ yields π and φ as in Proposition 7.

Next, we calculate the risk premium. Denote the rental price of capital by $R_{t,F}$ in the state of the fully-revealing equilibrium and by $R_{t,N}$ in the state of the non-revealing equilibrium. Suppose the current state in period t is the fully-revealing equilibrium. The bond yield is given by

$$
\begin{aligned}
\log R_{ft,F} &= \log \frac{q_{F,F}\mathbb{E}\left[R^{\rho}_{t+1,F}|k_t,a_t,a_{t+1}\right]+(1-q_{F,F})\,\mathbb{E}\left[R^{\rho}_{t+1,N}|k_t,a_t,a_{t+1}\right]}{q_{F,F}\mathbb{E}\left[R^{\rho-1}_{t+1,F}|k_t,a_t,a_{t+1}\right]+(1-q_{F,F})\,\mathbb{E}\left[R^{\rho-1}_{t+1,N}|k_t,a_t,a_{t+1}\right]} \\
&= \left\{\begin{array}{l}
\{\log\alpha+[1+(1-\alpha)\varphi]\,a_{t+1}+(1-\alpha)(\pi-1)\,k_{t+1}\} \\
+\log\dfrac{\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left[\exp\left[\rho(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t,a_{t+1}\right] \\ +(1-q_{F,F})\,\mathbb{E}\left[\exp\left(\rho(1-\alpha)\,n^{c,N}\right)|k_t,a_t,a_{t+1}\right]\end{array}\right\}}{\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left\{\exp\left[(\rho-1)(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t,a_{t+1}\right\} \\ +(1-q_{F,F})\exp\left[(\rho-1)(1-\alpha)\,n^{c,N}\right]\end{array}\right\}}
\end{array}\right\}.
\end{aligned}
$$

The expected capital return is given by

$$
\begin{aligned}
&\log\mathbb{E}\left(R_{t+1}|k_t,a_t,a_{t+1}\right) \\
=\;&\log\left\{q_{F,F}\mathbb{E}\left[\exp\left(r_{t+1,F}\right)|k_t,a_t,a_{t+1}\right]+(1-q_{F,F})\,\mathbb{E}\left[\exp\left(r_{t+1,N}\right)|k_t,a_t,a_{t+1}\right]\right\} \\
=\;&\left\{\begin{array}{l}
\log\alpha+[1+(1-\alpha)\varphi]\,a_{t+1}+(1-\alpha)(\pi-1)\,k_{t+1} \\
+\log\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left(\exp\left((1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right)|k_t,a_t,a_{t+1}\right) \\ +(1-q_{F,F})\exp\left[(1-\alpha)\,n^{c,N}\right]\end{array}\right\}
\end{array}\right\}.
\end{aligned}
$$

Hence, the risk premium in the state of the fully-revealing equilibrium is

$$
\begin{aligned}
&\log\mathbb{E}\left(R_{t+1}|k_t,a_t,a_{t+1}\right)-\log R_{ft,F} \\
=\;&\left\{\begin{array}{l}
\log\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left(\exp\left((1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right)|k_t,a_t,a_{t+1}\right) \\ +(1-q_{F,F})\exp\left[(1-\alpha)\,n^{c,N}\right]\end{array}\right\} \\
-\log\dfrac{\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left[\exp\left[\rho(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t,a_{t+1}\right] \\ +(1-q_{F,F})\exp\left[\rho(1-\alpha)\,n^{c,N}\right]\end{array}\right\}}{\left\{\begin{array}{l} q_{F,F}\mathbb{E}\left\{\exp\left[(\rho-1)(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t,a_{t+1}\right\} \\ +(1-q_{F,F})\exp\left[(\rho-1)(1-\alpha)\,n^{c,N}\right]\end{array}\right\}}
\end{array}\right\}. \qquad \text{(A.10)}
\end{aligned}
$$

Similarly, we can work out the risk premium in the state of the non-revealing equilibrium:

$$\begin{aligned}&\log \mathbb{E}\left(R_{t+1}|k_t,a_t\right)-\log R_{ft,N}\\ =&\left\{\begin{array}{l}(1-\rho)\left[1+(1-\alpha)\varphi\right]^2\sigma_a^2\\ +\log\left\{\begin{array}{l}q_{N,F}\mathbb{E}\left(\exp\left[(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t\right)\\ +(1-q_{N,F})\exp\left[(1-\alpha)n^{c,N}\right]\end{array}\right\}\\ -\log\dfrac{\left\{\begin{array}{l}q_{N,F}\mathbb{E}\left[\exp\left[\rho(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t\right]\\ +(1-q_{N,F})\exp\left[\rho(1-\alpha)n^{c,N}\right]\end{array}\right\}}{\left\{\begin{array}{l}q_{N,F}\mathbb{E}\left\{\exp\left[(\rho-1)(1-\alpha)\left(n^{c,F}+\phi a_{t+2}\right)\right]|k_t,a_t\right\}\\ +(1-q_{N,F})\exp\left[(\rho-1)(1-\alpha)n^{c,N}\right]\end{array}\right\}}\end{array}\right\}. \qquad \text{(A.11)}\end{aligned}$$

If $|q_{F,F}-q_{N,F}|$ is small enough, (A.11) is higher than (A.10). In fact, if $q_{F,F}=q_{N,F}$, the other terms are the same while the additional term $(1-\rho)\left[1+(1-\alpha)\varphi\right]^2\sigma_a^2$ in (A.11) is positive.

Proof of Corollary 4: We first solve the equilibrium of the OLG model with the Markov process of risk aversion. We conjecture that the labor decision rule is $n_{t,H}=n^{c,H}+\varphi a_t+\pi k_t+(\phi a_{t+1}+z_t)$ when workers in period t have risk aversion ρ_H (regarding their consumption in period $t+1$) and it is $n_{t,L}=n^{c,L}+\varphi a_t+\pi k_t+(\phi a_{t+1}+z_t)$ when workers in period t have risk aversion ρ_L. Let $n_{t,L}\equiv\log N_{t,L}$ and $n_{t,H}\equiv\log N_{t,H}$.

Suppose workers in current period t have risk aversion ρ_H (regarding their consumption in period $t+1$). Their labor decision in period t is

$$N_{t,H}^{(\gamma+1)-(1-\alpha)\alpha}=\frac{\alpha(1-\alpha)^\alpha}{\psi}\left[A_tK_t^\alpha\right]^\alpha\left\{\begin{array}{l}\xi_{H,H}\mathbb{E}\left[\left(A_{t+1}N_{t+1,H}^{1-\alpha}\right)^{\rho_H}|P_t,A_t,K_t\right]\\ +\left(1-\xi_{H,H}\right)\mathbb{E}\left[\left(A_{t+1}N_{t+1,L}^{1-\alpha}\right)^{\rho_H}|P_t,A_t,K_t\right]\end{array}\right\}^{\frac{1}{\rho_H}}.$$

Hence,

$$n_{t,H}=\left\{\begin{array}{l}\left[\alpha\theta+\theta(1-\alpha)\pi\right]a_t+\left[\alpha^2\theta+\theta(1-\alpha)\pi\alpha\right]k_t+\theta(1-\alpha)^2\pi n_{t,H}\\ +\frac{\theta}{\rho_H}\log\left\{\xi_{H,H}\exp\left[(1-\alpha)\rho_Hn^{c,H}\right]+\left(1-\xi_{H,H}\right)\cdot\exp\left[(1-\alpha)\rho_Hn^{c,L}\right]\right\}\\ +\theta(1-\alpha)\pi\log(1-\alpha)\\ +\theta\left[\left(-\frac{1}{2}\left[(1-\alpha)\phi\right]+\frac{1}{2}\rho_H\left[(1-\alpha)\phi\right]^2\right)\sigma_a^2+\frac{1}{2}\rho_H\left[(1-\alpha)\sigma_z\right]^2\right]\\ +\frac{\theta}{\rho_H}\log\mathbb{E}\left[\exp\left(\rho_H\left[1+(1-\alpha)\varphi\right]a_{t+1}\right)|\phi a_{t+1}+z_t,a_t\right]\end{array}\right\}.$$

Comparing terms with $n_{t,H}=n^{c,H}+\varphi a_t+\pi k_t+(\phi a_{t+1}+z_t)$ yields φ, π and ϕ as in Proposition 7.

Next, we work out the risk premium. Denote the rental price of capital by $R_{t+1,H}$ in period $t+1$ when workers in period $t+1$ have risk aversion ρ_H and by $R_{t+1,L}$ when workers in period

$t+1$ have risk aversion ρ_L. The bond yield in the state of $\rho = \rho_H$ is

$$
\begin{aligned}
& \log R_{ft,H} \\
= \; & \log \frac{\xi_{H,H}\mathbb{E}\left[\exp\left(\rho_H r_{t+1,H}\right)|\phi a_{t+1}+z_t\right] + \left(1-\xi_{H,H}\right)\mathbb{E}\left[\exp\left(\rho_H r_{t+1,L}\right)|\phi a_{t+1}+z_t\right]}{\xi_{H,H}\mathbb{E}\left[\exp\left(\left(\rho_H-1\right) r_{t+1,H}\right)|\phi a_{t+1}+z_t\right] + \left(1-\xi_{H,H}\right)\mathbb{E}\left[\exp\left(\left(\rho_H-1\right) r_{t+1,L}\right)|\phi a_{t+1}+z_t\right]} \\
= \; & \left\{ \begin{array}{l} \mathbb{E}\left\{ \begin{array}{l} \log\alpha + \left[1+(1-\alpha)\varphi\right] a_{t+1} + (1-\alpha)(\pi-1) k_{t+1} \\ +(1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)|\phi a_{t+1}+z_t, a_t \end{array} \right\} \\ +\frac{1}{2}\left(2\rho_H-1\right)\cdot Var\left\{\left[1+(1-\alpha)\varphi\right] a_{t+1} + (1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)|\phi a_{t+1}+z_t, a_t\right\} \\ +\log \frac{\xi_{H,H}\exp\left[\rho_H(1-\alpha)n^{c,H}\right]+\left(1-\xi_{H,H}\right)\exp\left[\rho_H(1-\alpha)n^{c,L}\right]}{\xi_{H,H}\exp\left[(\rho_H-1)(1-\alpha)n^{c,H}\right]+\left(1-\xi_{H,H}\right)\exp\left[(\rho_H-1)(1-\alpha)n^{c,L}\right]} \end{array} \right\}.
\end{aligned}
$$

The expected capital return is given by

$$
\begin{aligned}
& \log \mathbb{E}\left[\exp\left(r_{t+1}\right)|k_t, a_t, \phi a_{t+1}+\sigma_z z_t\right] \\
= \; & \left\{ \begin{array}{l} \mathbb{E}\left\{ \begin{array}{l} \log\alpha + \left[1+(1-\alpha)\varphi\right] a_{t+1} + (1-\alpha)(\pi-1) k_{t+1} \\ +(1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)|\phi a_{t+1}+\sigma_z z_t, a_t \end{array} \right\} \\ +\frac{1}{2} Var\left\{\left[1+(1-\alpha)\varphi\right] a_{t+1} + (1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)|\phi a_{t+1}+z_t, a_t\right\} \\ +\log\left\{\xi_{H,H}\exp\left[(1-\alpha)n^{c,H}\right] + \left(1-\xi_{H,H}\right)\exp\left[(1-\alpha)n^{c,L}\right]\right\} \end{array} \right\}.
\end{aligned}
$$

Hence, the risk premium in the state of $\rho = \rho_H$ is

$$
\begin{aligned}
& \log \mathbb{E}\left(R_{t+1}|\phi a_{t+1}+\sigma_z z_t, a_t, \rho=\rho_H\right) - \log R_{ft,H} \\
= \; & \left\{ \begin{array}{l} \left(1-\rho_H\right) Var\left\{\left[1+(1-\alpha)\varphi\right] a_{t+1} + (1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)|\phi a_{t+1}+z_t, a_t\right\} \\ +\log\left\{\xi_{H,H}\exp\left[(1-\alpha)n^{c,H}\right] + \left(1-\xi_{H,H}\right)\exp\left[(1-\alpha)n^{c,L}\right]\right\} \\ -\log\left\{\frac{\xi_{H,H}\exp\left[\rho_H(1-\alpha)n^{c,H}\right]+\left(1-\xi_{H,H}\right)\exp\left[\rho_H(1-\alpha)n^{c,L}\right]}{\xi_{H,H}\exp\left[(\rho_H-1)(1-\alpha)n^{c,H}\right]+\left(1-\xi_{H,H}\right)\exp\left[(\rho_H-1)(1-\alpha)n^{c,L}\right]}\right\} \end{array} \right\}.
\end{aligned}
\tag{A.12}
$$

Similarly, the risk premium in the state of $\rho = \rho_L$ is (A.12) with ρ_H being replaced by ρ_L and $\xi_{H,H}$ being replaced by $\xi_{L,H}$.

Therefore, the difference (variation) in the risk-premium between state $\rho = \rho_L$ and $\rho = \rho_H$ is

$$
\begin{aligned}
& \left[\log \mathbb{E}\left(R_{t+1}|\phi a_{t+1}+\sigma_z z_t, \rho=\rho_L\right) - \log R_{ft,L}\right] - \left[\log \mathbb{E}\left(R_{t+1}|\phi a_{t+1}+\sigma_z z_t, \rho=\rho_H\right) - \log R_{ft,H}\right] \\
= \; & \left\{ \begin{array}{l} \left(\rho_H-\rho_L\right) Var\left\{\left[1+(1-\alpha)\varphi\right] a_{t+1} + (1-\alpha)\left(\phi a_{t+2}+z_{t+1}\right)|\phi a_{t+1}+z_t, a_t\right\} \\ +\left(\begin{array}{c} \log\left\{\xi_{L,H}\exp\left[(1-\alpha)n^{c,H}\right] + \left(1-\xi_{L,H}\right)\exp\left[(1-\alpha)n^{c,L}\right]\right\} \\ -\log\left\{\xi_{H,H}\exp\left[(1-\alpha)n^{c,H}\right] + \left(1-\xi_{H,H}\right)\exp\left[(1-\alpha)n^{c,L}\right]\right\} \end{array} \right) \\ +\left[f\left(\rho_H;\xi_{H,H}\right) - f\left(\rho_L;\xi_{L,H}\right)\right] \end{array} \right\},
\end{aligned}
\tag{A.13}
$$

where

$$f(\rho;\xi)=\log\frac{\xi\exp\left[\rho(1-\alpha)n^{c,H}\right]+(1-\xi)\exp\left[\rho(1-\alpha)n^{c,L}\right]}{\xi\exp\left[(\rho-1)(1-\alpha)n^{c,H}\right]+(1-\xi)\exp\left[(\rho-1)(1-\alpha)n^{c,L}\right]}.$$

In (A.13), we prove that the sum of the second term and the third term is positive when $\left|\xi_{L,H}-\xi_{H,H}\right|$ is small enough. To see this, if $\xi_{L,H}=\xi_{H,H}=\xi$, the second term becomes 0 and the third term becomes $f(\rho_H;\xi)-f(\rho_L;\xi)>0$ by

$$sgn\left(\frac{\partial f(\rho;\xi)}{\partial\rho}\right)=sgn\left(\left\{\exp\left[(1-\alpha)n^{c,H}\right]-\exp\left[(1-\alpha)n^{c,L}\right]\right\}\left(n^{c,H}-n^{c,L}\right)\right)=1.$$

The first term in (A.13) is positive and increasing in $\rho_H-\rho_L$ and σ_z^2. Note that in Corollary 1, we have proved that the first term is increasing in σ_z^2 when ϕ is decreasing in σ_z^2. Overall, (A.13) is positive and increasing in σ_z^2 given ρ_H and ρ_L.

B Asymmetric Sentiment-driven Equilibrium in Section 5.2

Here we construct an asymmetric sentiment-driven equilibrium that exhibits contagion. For $\ell=0$,

$$\log P_{00}=\begin{cases}\log\alpha+(1-\alpha)\theta(g+a_{20}) & \text{if } g\geq 0\\ \log\alpha+(1-\alpha)\theta\log\left[\frac{\exp(g)+\exp(z_0)}{2}\right] & \text{if } g<0\end{cases}$$

and

$$\log N_{10}=\begin{cases}\theta(g+a_{2\ell}) & \text{if } g\geq 0\\ \theta\log\left[\frac{\exp(g)+\exp(z_0)}{2}\right] & \text{if } g<0\end{cases},$$

where z_0 is the sentiment shock in country 0 with p.d.f. as

$$f(z_0)=\begin{cases}0 & \text{if } z\geq 0\\ \frac{1}{\Phi(\frac{1}{2}\sigma_g)}\cdot\frac{1}{\sigma_g\sqrt{2\pi}}e^{-\frac{(z_0+\sigma_g^2/2)^2}{2\sigma_g^2}} & \text{if } z<0\end{cases};$$

for $\ell>0$,

$$p_{0\ell}=\log P_{0\ell}=\begin{cases}\log\alpha+(1-\alpha)\theta(g+a_{2\ell}) & \text{if } g\geq 0\\ \log\alpha+(1-\alpha)\theta\log\left[\frac{\exp(g)+\exp(z_0)}{2}\right]+(1-\alpha)(\phi a_{2\ell}+z_\ell) & \text{if } g<0\end{cases}$$

and

$$\log N_{1\ell}=\begin{cases}\theta(g+a_{2\ell}) & \text{if } g\geq 0\\ \theta\log\left[\frac{\exp(g)+\exp(z_0)}{2}\right]+(\phi a_{2\ell}+z_\ell) & \text{if } g<0\end{cases},$$

with $\phi=\frac{\theta}{2}\pm\frac{\sqrt{\theta^2\sigma_\alpha^2-4\sigma_z^2}}{2\sigma_\alpha}$, where $z_\ell\sim N\left(0,\sigma_z^2\right)$ is the sentiment shock in country $\ell>0$.

The workers in each country can infer whether $g\geq 0$ or $g<0$ by looking at the dispersion of asset prices across countries $\ell > 0$. Notice that dispersion of $p_{0\ell}$ is $SD(p_{0\ell}) = (1-\alpha)\theta\sigma_a$ if $g \geq 0$ and $SD(p_{0\ell}) = (1-\alpha)\sqrt{\phi^2\sigma_a^2 + \sigma_z^2}$ if $g < 0$. Since $\frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2} = 1$ and $\phi \leq \theta$, we have $(1-\alpha)\sqrt{\phi^2\sigma_a^2 + \sigma_z^2} \leq (1-\alpha)\theta\sigma_a$.

In the extreme case of $\sigma_z = 0$ and $\phi = 0$, the above equilibrium becomes

$$\log P_{0\ell} = \begin{cases} \log\alpha + (1-\alpha)\theta(g + a_{2\ell}) & \text{if } g \geq 0 \\ \log\alpha + (1-\alpha)\theta\log\left[\frac{\exp(g)+\exp(z_0)}{2}\right] & \text{if } g < 0 \end{cases}$$

and

$$\log N_{0\ell} = \begin{cases} \theta(g + a_{2\ell}) & \text{if } g \geq 0 \\ \theta\log\left[\frac{\exp(g)+\exp(z_0)}{2}\right] & \text{if } g < 0 \end{cases}. \tag{B.1}$$

for $\ell \geq 0$.

In such an equilibrium, when $g \geq 0$, the equilibrium is fully-revealing; when $g < 0$, the financial prices across countries $\ell > 0$ are perfectly synchronized.[25] When workers in each country observe perfectly synchronized asset prices, they infer that $g < 0$ with probability 1; their information extraction leads to their labor supply as in (B.1). The intuition behind the above equilibrium is the following. Investors in the financial market in country 0 as well as in country $\ell > 0$ "overweight" the impact of the *global* shock g when $g < 0$. The extreme case helps highlight the intuition more sharply. In an economic downturn, investors perceive that the global economy is fully synchronized and local shocks do not matter. Under the information frictions and the feedback effect, investors' perception of synchronization leads to actual synchronization. The financial prices and the real output in period 1 across countries become fully synchronized. In this sense, sentiments in financial markets can *amplify* the cross-country comovement because country-specific shocks are "ignored" by the investors.

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