Short-selling Attacks and Creditor Runs*

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Abstract

This paper investigates the mechanism through which short selling of a bank's stocks can trigger the failure of the bank. In the model, creditors, who learn information from stock prices, will grow increasingly unsure about the bank's true fundamentals in facing noisier stock prices; thus a run on the bank is more likely because of creditors' concave payoff. Understanding this, speculators conduct short selling beforehand to amplify (il)liquidity and add noise to stock prices, triggering a bank run, and subsequently profit from the bank's failure. We show that short-selling attacks on a bank involve two runs: the aggressive run among speculators and the conservative run among creditors. These two runs interact and reinforce each other, with compound feedback loops that drastically increase the probability of the collapse of the bank. We discuss policy implications of the model.

JEL classification: G01; G20, G14; G18; D82; D84

Keywords: Short-selling attacks, Creditor runs, Coordination, Information asymmetry, Feedback

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We must understand financial markets through a new paradigm which recognizes that they always provide a biased view of the future, and that the distortion of prices in financial markets may affect the underlying reality that those prices are supposed to reflect. I call this feedback mechanism "reflexivity."

George Soros (2009)

1 Introduction

There have been tough regulations on short selling recently, in particular short selling of financial stocks, which has been blamed for playing an important and direct role in destabilizing the financial system.

Short selling of stocks has been accused of constituting a direct cause of the collapses of Bear Stearns and Lehman Brothers. In late 2008, several countries, including the US and UK, imposed an emergency order to ban the short selling of financial stocks. In executing the short-sale ban order (Securities Exchange Act NO. 34-58592 / September 18, 2008),[1] the SEC concluded:

Short selling in the securities of a wider range of financial institutions may be causing sudden and ***excessive fluctuations*** *of the prices of such securities in such a manner so as to threaten fair and orderly markets.*

Similarly, short selling has been accused of worsening the European sovereign debt crisis. On Aug 12, 2011, in a new wave of the European debt crisis, after Greece, four other EU nations (France, Italy, Spain and Belgium) banned short selling on bank stocks, in order to "calm volatility".[2]

The regulation of bans on short selling, however, is extremely controversial in the financial industry as well as with academics. In fact, the empirical research on this topic is not conclusive.[3]

Our paper examines this question theoretically. We investigate the specific mechanism through which short selling can trigger bank failures and, in particular, addresses the question of why banks (as opposed to standard corporations) are unique in suffering short-selling attacks.

[1] http://www.sec.gov/rules/other/2008/34-58592.pdf

[2] http://money.cnn.com/2011/08/12/news/international/short_selling_ban/index.htm

[3] Beber and Pagano (2013) and Boehmer et al. (2013) find that short-sale bans led to reductions in market liquidity and slower price discovery. However, Arce and Mayordomo (2012) find that in the Spanish stock market the ban helped stabilize the probability of default of medium-sized banks and that the volatility of the stock return of these banks fell significantly after the ban. Pais and Stork (2012) document that short selling increased the volatility and contagion risk of financial institutions in the four EU nations mentioned above.

In the literature, a few theoretical papers give some important insights about the destabilizing effects of short selling. Concretely, a lower asset price caused by short selling can generate a spillover effect on decision makers, through negative information learned from the lower asset price about fundamentals (Goldstein and Guembel (2008), Goldstein, Ozdenoren and Yuan (2013)) or through funding liquidity constraints (Brunnermeier and Oehmke (2013)). Our paper offers a related but different insight. In our paper, spillovers arise through the higher uncertainty about the fundamentals: the effect is on the *variance* rather than on the perceived *mean* of fundamentals.

Specifically, in our model, the danger of short-selling attacks is that they can reduce *price informativeness*, leading to an increase in uncertainty and information asymmetry about the bank's true fundamental value. In the model, speculators conduct short selling in the stock market with stochastic market liquidity. When the market is not liquid enough, speculators can influence and depress the stock price. When the market is very liquid, the selling by speculators is "washed out" and cannot influence the stock price much. In the presence of imperfect information about fundamentals, creditors use the stock price to learn (in a Bayesian fashion) about the true fundamental value. They will grow increasingly unsure about the true fundamental value in the face of a low stock price. A low price may be due to short-selling attacks or to bad fundamentals. In a rational-expectations equilibrium, creditors can still conjecture the true fundamental value correctly, on average, by observing the stock price; however, the precision of their conjecture decreases. That is, for a given stock price, both the downside and the upside risk in estimating the true fundamental value increase. Considering that the debt-contract payoff for creditors is *concave*, they react to the increase in downside risk by running on the bank more often, which can eventually lead to the bank's failure. In short, the main mechanism of the model can be described as follows: (Step 1) Speculative short selling increases noise in the price signal; (Step 2) this lowers the expected payoff for the creditors with higher uncertainty (less information) about the fundamentals; (Step 3) there is a larger withdrawal among the creditors; (Step 4) this in turn leads to more incentives to short among the speculators.[4]

We believe that the mechanism presented above is in line with what happened in reality. In fact, in both the SEC's official document and the European Securities and Market Authority (ESMA)'s statement,[5] the key words '*fluctuation*' and '*volatility*' appear. These terms, to a large extent,

[4] We owe this description to one referee.

[5] See ESMA/2011/266.

confirm that short selling increases uncertainty. The economic translation of those terms in our model is 'information asymmetry'. The evidence in Arce and Mayordomo (2012) and Pais and Stork (2012) also indicates that short selling increased the banks' stock price volatility.

Our model further answers two other critical questions regarding short-selling attacks on banks. Namely, how can dispersed speculators form collective force to be able to move stock prices significantly (especially considering that banks are typically large-cap stocks), and why do speculators choose to attack banks rather than standard corporations?

We argue that short-selling attacks on a bank involve two coordination problems or *two runs*: the aggressive run among speculators and the conservative run among creditors. The first run is the coordination that is formed among speculators (in the spirit of Morris and Shin (1998)). The second run is the classic creditor run (in the spirit of Diamond and Dybvig (1983)). We show that the two runs interact with each other. The speculator run influences *information* of the creditor run while the creditor run determines the *payoffs* of the speculator run.

Concretely, our model features two-stage global games. In the first stage, short sellers take positions (the speculator run). In the second stage, creditors decide whether or not to roll over their debt (the creditor run). We demonstrate the feedback between the two stages. We first abstract from the speculator run and focus on the second stage of the game - the creditor run. We show an intuitive result: creditor runs are more easily triggered when stock prices are noisier. In fact, when the stock price is very noisy and volatile, creditors have no clue about the bank's true fundamentals. In facing a low stock price, creditors understand that it may be for some exogenous reasons, in which case the stock is underpriced. But it can also be due to true bad fundamentals. Because of the asymmetry in payoffs between the upside risk and the downside risk for the debt contract, creditors react to the noisier stock price by running on the bank at a higher threshold.

We then move to the first stage of the game and show that endogenous short-selling attacks by speculators arise. That is, speculators have incentives to coordinate automatically to create noise in the stock price (by shorting). In other words, there exist *endogenous* strategic complementarities among speculators in the short-selling attack. For a particular individual speculator, the more aggressive other speculators are in short selling, the noisier the stock price is; a noisier stock price leads to a higher likelihood of a creditor run, which translates into a higher probability of success for the short-selling attack. Therefore, if other speculators are aggressive in short selling, in equilibrium it is optimal for an individual speculator to be aggressive as well.

We show that the two runs not only interact but also reinforce each other, with the result of drastically increasing the probability of the collapse of the bank. That is, there is a two-way feedback loop. We have already discussed one direction of the feedback – the speculator run impacts the creditor run (i.e., through reducing the stock price informativeness). The other way of the feedback reflects how the creditors' actions affect the speculator run. We illustrate two channels of forces: if the creditors decline to roll over at a higher fundamental value, this not only increases the success probability of the speculators' attack but also enhances the gain for the speculators in case of a successful attack.

The model offers two cross-sectional predictions. First, firms with lower fundamentals are more likely to be subject to short-selling attacks, and are more likely to fail. Second, for a given fundamental value, firms with a higher degree of maturity mismatch in their balance sheets are more likely to incur short-selling attacks. Indeed, we show that due to the compound reinforcing spirals, a slight difference in maturity mismatch can make a large difference in the probability of incurring short-selling attacks. This implies that short-selling attacks should be much more likely on financial firms, typically with much high maturity mismatch than standard corporations.

Interestingly, our paper shows the asymmetry between short-side and long-side speculation.[6] In our model, both long-side and short-side attacks increase uncertainty and thereby make a creditor run more likely. However, speculators need to gain from their positions. A short position can make profits only when the price goes down while a long position can gain only when the price goes up. Because a creditor run leads to a fall in stock prices, speculators *optimally* choose short-side attacks.

Related literature. Although short selling stabilizes prices most of the time, we study the destabilizing effects of short selling at times of market turbulence. We show that when a bank is sufficiently weak in fundamentals and is close to bankruptcy, it can attract short-selling attacks, in which case short selling can generate a negative spillover effect on the *real* side – the bank's business operation (i.e., financing). Our paper studies the impact of short selling on financial stability beyond the functioning of the stock market.

Our paper is related to the theoretical literature on short selling. Goldstein and Guembel (2008) study a model where firm managers learn from the price about real investment opportunities and

[6] The asymmetry of speculation is also shown in Goldstein and Guembel (2008) and Edmans, Goldstein and Jiang (2011).

speculators' trading can thus distort managers' investment decisions.[7] The authors show that the presence of such feedback creates an incentive for an uninformed trader to sell the firm's stock. Goldstein, Ozdenoren and Yuan (2013) show strategic complementarities among speculators as a result of the feedback between financial market speculations and the real investment decision. Our paper complements these papers by focusing on why short-selling attacks are particularly prominent on financial firms and offering the explanation of double runs. In our model, the manipulation happens because of the impact on creditors' financing (rollover) decisions rather than on managers' real-investment decisions. Brunnermeier and Pedersen (2005) study predatory trading, showing that predator traders may deliberately dump the asset that a distressed trader holds, depressing the asset's price and triggering the trader's constraints to bind (e.g., margin calls). Their model captures well situations of predatory trading in asset markets in which traders like hedge funds typically face some external constraints, such as mark-to-market based margin calls from their brokers. In the same vein, Brunnermeier and Oehmke (2013) show that predatory short selling of a financial institution's stock can emerge in equilibrium when there are leverage constraints imposed by short-term creditors. In our paper, we offer the insight that it is the change in stock price informativeness, rather than the stock price itself, that explains why short selling can trigger the failure of a bank.

The model in our paper presents Morris-Shin (1998) meeting Diamond-Dybvig (1983). In the model, two groups of investors move *sequentially*. The first group of investors coordinates to conduct short-selling attacks, with the objective of triggering the conservative run of the second group. We show that there exists a two-way feedback between the actions of these two groups, and that the feedback generates self-reinforcing spirals. We endogenize the strategic complementarities, the information structure and the payoff structure of the global games in our model.

Our paper is related to Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2006) in that all papers endogenize public information of global games. The main focus of these papers is on studying equilibrium multiplicity, while our paper emphasizes the comparative statics of equilibrium and the interaction between two runs. There are two runs instead of one run in our

[7] Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren and Yuan (2011) derive endogenous complementarities as a result of learning from the aggregate actions of agents. The focus of these papers is on over-weighting of public information, with the consequence of excessive non-fundamental volatility.

model, which is different from their papers.[8] Goldstein (2005) uses global games to model twin crises - the banking crisis and the currency crisis - and studies their interaction. In Goldstein (2005) the interaction between the two markets is not through the information channel.

Bernardo and Welch (2004) and Morris and Shin (2004) study market runs. These papers show in a rational framework that investors can rush to sell in financial markets. In some sense, our paper shows the interaction between market runs and bank runs. In our model, there are two stages: the first stage is similar to market runs and the second stage is bank runs. Our focus is on the two-way feedback between the two stages.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses predictions and policy implications of the model. Section 4 concludes.

2 Model

2.1 Model setup

The model has three dates: T_0, T_1 and T_2. There is no discount factor between T_1 and T_2. We discuss the three types of agents in our model in order: bank, speculators and creditors.

2.1.1 The bank

Consider a bank (investment or commercial bank) which holds one unit of asset. The asset, denoted by A, realizes a random cash flow $\widetilde{\theta} = \theta + \widetilde{\epsilon}$ at time T_2, where $\widetilde{\epsilon}$ is normally distributed as $\widetilde{\epsilon} \sim N(0, \sigma^2)$. The term θ represents the fundamentals of the bank. We assume that θ has an improper uniform prior over the real line. The value of θ is realized at T_0 but no one knows its realization, while the uncertainty of $\widetilde{\epsilon}$ is resolved at T_2. The bank finances its asset with short-term debt and equity. The short-term debt is the borrowing from a continuum of lenders with unit mass. The debt is short-term in the sense that the creditors have the right to decide at T_1 whether to roll over their lending or not. If a creditor declines to roll over, her claim is the face value of debt at T_1, denoted by F. If a creditor rolls over, her claim is the face value of debt plus interest, amounting to a total value of K at T_2 , where $K > F$. For the equity, we assume that the bank has a single unit of divisible share outstanding.

[8] The information structure is endogenous also in Angeletos, Hellwig and Pavan (2006, 2007) and Dasgupta (2007).

2.1.2 Speculators

There is a continuum of risk-neutral speculators with unit mass. At T_0, these speculators receive private information regarding θ. The information is imperfect. Specifically, speculator i observes a noisy signal $\theta^i = \theta + \omega^i$, where ω^i is normally distributed $\omega^i \sim N(0, \delta^2)$ and the noises are independent across speculators. Based on the private information received, a speculator decides whether to short sell the bank's stock at T_0 or not: a speculator can only short sell one unit of stock or decide not to take any position.

If a bank run occurs at T_1 and consequently the bank fails, it means that *the short-selling attack is successful*. A speculator's payoff (gain) in a successful short-selling attack is t. In contrast, if the bank does not fail at T_1, it means *the short-selling attack is unsuccessful*, in which case a speculator's payoff (loss) is r. At this stage, we assume that both t and r are exogenous and constant. We will endogenize t and r later. The fixed cost of conducting short-selling (e.g., margin or opportunity cost), whether the short-selling attack is successful or not, is c.

Speculator i's decision rule at T_0 is a map:

$$\theta^i \longmapsto (\text{Short sell, Not}),$$

where θ^i is speculator i's information and (Short-sell, Not) is her decision set.

2.1.3 Stock market

We use a stock market microstructure similar to that in Goldstein, Ozdenoren and Yuan (2013) (also see Hellwig, Mukherji and Tsyvinski (2006) and Albagli, Hellwig and Tsyvinski (2013)). There are two groups of traders in the stock market: strategic traders (i.e., speculators) and unsophisticated investors. At T_0, based on her private information, each speculator submits a market order of selling one or zero unit of stock to a Walrasian auctioneer. The Walrasian auctioneer then obtains the aggregate supply by speculators and also a noisy aggregate demand curve (schedule) from unsophisticated traders, and sets a price to clear the market. The noisy aggregate demand curve from unsophisticated traders is given by $Y(p) = \frac{v-p}{d}$. Let s represent the aggregate short selling by the speculators at T_0.[9] Then, by $s = Y$, the stock price at T_1 is given by $p = v - ds$.[10]

[9] The speculators borrow stocks from brokers. Brokers in turn use their clients' stocks to lend. The final lenders of stocks are therefore the existing shareholders, who are perhaps also unsophisticated 'buy and hold' investors.

[10] See, e.g., Grossman and Miller (1988), Bernardo and Welch (2004), Plantin et al. (2008) in the literature for the downward-sloping demand curve of stocks. Shleifer (1986) and others document the evidence.

The above market microstructure can be considered as a modified Kyle model. As in Kyle (1985), strategic traders submit their orders based on only their private signals, not contingent on and not learning from the market price. As Goldstein, Ozdenoren and Yuan (2013) write, "We assume that speculators do not observe the price when they trade and, hence they submit market orders, as in Kyle (1985). This setup of the financial market is a simple way to capture the (important) idea that speculators, when they trade, do not have the market information ..."

Our specification of the demand curve from unsophisticated traders can be justified with the microfoundation much like that in Brunnermeier and Pedersen (2005) (also see Grossman and Miller (1988)). Unsophisticated investors are long-term investors, who are *passive* and are *price-takers.* They do not have sufficient information, skills, or time to predict short-term events such as the probability of a bank run at T_1. Their demand is only based on long-term fundamental information. They are 'buy and hold' investors. The demand from an individual investor among them is $y^i(p) = \frac{v^i - p}{d_L}$, where v^i is the individual's private information regarding the long-term fundamentals of the stock, p is the stock price, and d_L measures risk aversion and other parameters.[11] Hence, if the size of the long-term investors sector is λ, the aggregate net demand from these investors is $Y(p) = \lambda \int y^i(p) = \lambda \frac{v-p}{d_L}$, which is a function of the true long-term fundamentals of the stock, v, as noises of private information cancel out in aggregating. Letting $d \equiv \frac{d_L}{\lambda}$, we have that $Y(p) = \frac{v-p}{d}$, where d measures the market liquidity (depth).

We assume that the market liquidity d is *random.* Pastor and Stambaugh (2003) and others document time-varying random market liquidity. Based on $d \equiv \frac{d_L}{\lambda}$, randomness in d can be due to either randomness in d_L (i.e., time-varying risk aversion) or randomness in λ. The intuition for the latter is as follows. There are times when a large number (a high λ) of long-term investors are able to provide liquidity to the market (the selling by speculators in this case is "washed out", like *a pebble being thrown into the ocean*) and consequently d is small, whereas there are times when the opposite is true and consequently d is high. We will specify the probability distribution of d later. Basically, randomness of d implies that speculators face *market liquidity risk* in short-selling attacks.

As v is the stock's long-term fundamentals, which in our model are equity value $E[\max(0, \widetilde{\theta} - K)|\theta]$, we have $v \equiv e(\theta) = E[\max(0, \widetilde{\theta} - K)|\theta]$. Therefore, the stock price at T_1 is given by

[11] Those investors are passive and price-taking, and do not learn information from the stock price.

$p = e(\theta) - ds$, a function of θ.[12]

In order to have a clean analysis and to highlight the core mechanism of the model, we use reduced-form equity value: $v \equiv e(\theta) = E[\widetilde{\theta} - K|\theta]$. So $e(\theta) = \theta - K$. Hence, $p = \theta - K - ds$. The mechanism of the model does not depend on this simplification. In the appendix, we give an illustration showing that as long as $e(\theta)$ is increasing and weakly convex in θ, the robustness of the model is intact. We denote $P \equiv p + K$, where P is interpreted as the *stock price-based asset value* of the bank. Therefore, we have $P = \theta - ds$.

The equation $P = \theta - ds$ is the central result in this subsection. Basically, as the stock price contains information about the asset value, investors can use the stock price to infer the asset value. The stock price-based asset value of the bank reflects two elements: the true fundamentals of the asset, θ, and the short-selling impact, ds.

Discussions of the stock market setup In our model, we can think that there are two types of trading: trading by the unsophisticated long-term investors and trading by the speculators. The first type increases information on the asset price with information aggregation (as in Grossman and Stiglitz (1980) and Angeletos and Werning (2006)) while the second type decreases information (in the spirit of Grossman and Miller (1988)). The focus of our paper is on the second type – the destabilizing effect of the trading of speculators.

The extant literature often shows that speculations, including short selling, can increase price informativeness, while in our model the opposite is true. The difference originates in the existence of feedback from the *real* side in our model. Specifically, in our model, when a firm (bank) is weak in fundamentals and is close to bankruptcy, the *collective* force of speculators can trigger the problem on the real side (i.e., a creditor run) and change the stock fundamentals. This in turn induces the speculators to trade in the *same* direction (strategic complementarities),[13] which reduces price informativeness.

[12] Similar to Brunnermeier and Pedersen (2005), the stock price is not forward-looking in incorporating the probability of short-term bank failure arising from a bank run at T_1 (i.e., the probability of a run at T_1). This is because the long-term investors - the market markers - are unsophisticated and do not understand the short-term events.

[13] This 'coordination' motive can lead to limits to arbitrage (also see Abreu and Brunnermeier (2002), Ozdenoren and Yuan (2008), Goldstein et al. (2013), and Brunnermeier and Nagel (2004)).

2.1.4 Creditors and bank run

Creditors are risk-neutral. At T_1, all creditors observe the stock price p, equivalent to knowing P. In addition, they receive private information regarding θ. Creditor j's private signal is $\theta^j = \theta + \varsigma^j$, where ς^j is normally distributed as $\varsigma^j \sim N(0, \gamma^2)$. Based on both stock price information and the private signal, each creditor needs to decide whether to roll over her lending. That is, creditor j's decision at T_1 is a map:

$$(\theta^j, P) \longmapsto (\text{Rollover, Not}).$$

We assume that the asset of the bank is illiquid. The liquidation value of asset A at T_1 is L, where $L < F$. Therefore, if more than $\frac{L}{F}$ proportion of creditors declines to roll over at T_1, the liquidation value is not sufficient to cover the creditors' claims, and consequently *the bank fails.* Alternatively, one may think of L as the collateral value of the bank's asset. This means that the bank can raise at most L amount of cash at T_1 by using its asset as collateral. If the demand of cash exceeds L at T_1, the bank fails. This is the classic bank run problem. We use the term L (for a given F) to measure the maturity (liquidity) mismatch of the balance sheet of the bank: the lower the L, the more severe the maturity (liquidity) mismatch.

To fully describe the payoff structure of creditors in a bank run, we need to write the payoff of an individual creditor to roll over (vs. not roll over) as a continuous function of the total number of creditors who roll over. For the payoff structure of a bank run with continuous states, we refer the reader to Dasgupta (2004), Goldstein and Pauzner (2005) and Liu and Mello (2012). For our purposes, however, it is sufficient to use the discrete-state setup, which follows the work of Morris and Shin (2009) and Rochet and Vives (2004).

	Less than $\frac{L}{F}$ proportion withdrawing (i.e., bank survives)	More than $\frac{L}{F}$ proportion withdrawing (i.e., bank fails)
Roll over	$\min(\widetilde{\theta}, K)$ (at T_2)	0 (at T_2)
Withdraw	F (at T_1)	F (at T_1)

Table 1: Creditor-run payoff structure

The payoff matrix in Table 1 can be interpreted in the following way, as in Morris and Shin (2009). If less than $\frac{L}{F}$ proportion of creditors withdraws, the bank does not fail. The bank regains its initial status after the failed run. In this case, the repayment to a creditor is not affected: the payoff is the *notional* value K in the case of rolling over, and F in the case of withdrawing.[14] In contrast, if more than $\frac{L}{F}$ proportion of creditors withdraws, the bank fails. A creditor who rolls over is left with nothing, whereas a creditor who withdraws has a payoff of F (due to outside investment opportunities).[15] The above interpretation is equivalent to that in Rochet and Vives (2004), where a third party, fund managers, is introduced.

The above payoff structure has the advantage of being simple and avoiding technique complications, and at the same time sufficing to capture the key feature of a creditor-run game — the (global) strategic-complementarities payoffs. That is, if more than $\frac{L}{F}$ proportion of creditors withdraws, the optimal strategy for an individual creditor is 'withdrawing'. If less than $\frac{L}{F}$ proportion of creditors withdraws, the optimal strategy is likely to be 'rolling over'.

Figure 1 summarizes the main setup of the model.

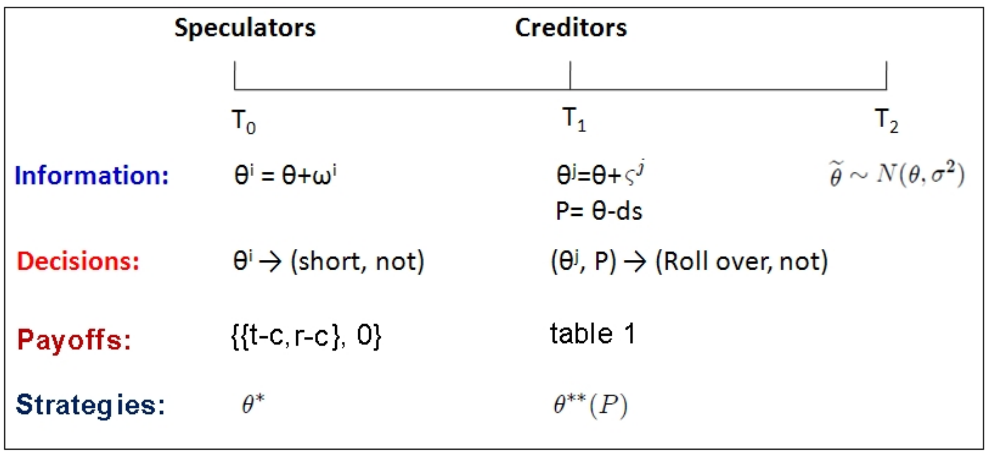


Figure 1: Main setup of the model

2.2 A simple example for the mechanism of the model

In order to facilitate the analysis in the next subsection, we use a simple example to illustrate the key mechanism of the model.

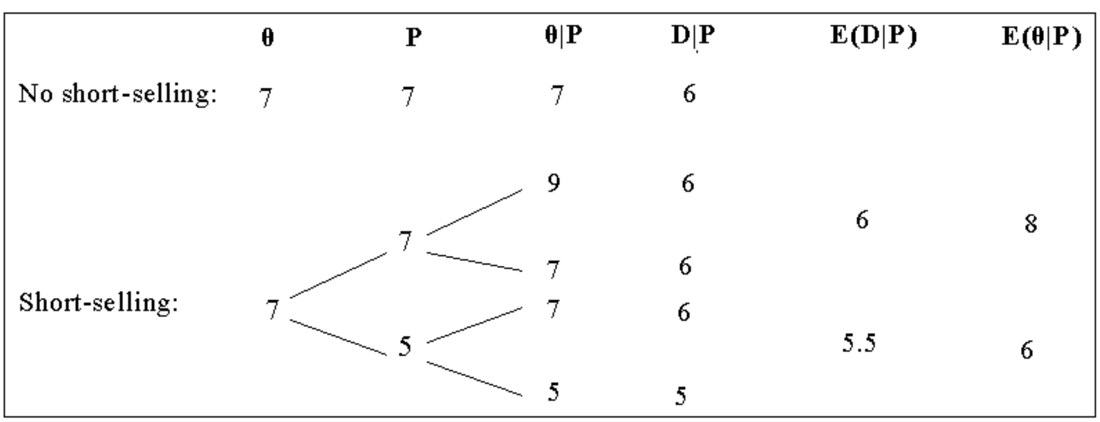
[14]For tractability, we assume that the debt contract stipulates that an individual staying creditor is paid at T_2 according to the total proportion of staying creditors being 1 and the equityholders obtain residual payoffs. This assumption is not necessary for the limiting case of $\gamma \rightarrow 0$ for given h.

[15]If more than $\frac{L}{F}$ proportion of creditors withdraws and thus the bank fails, the payoff for a creditor who withdraws should be slightly less than the face value F as the total liquidation value is L, less than F. However, as long as the payoff is higher than 0, the optimal strategy is 'withdrawing'. Thus, when the payoff is written as F, the nature of (global) strategic complementarities does not change.

Specifically, we assume that d follows the simple two-state Bernoulli distribution: d can be either 0 or 1 with 50% probability each. The economic interpretation is as follows. The long-term investors as market makers are sometimes very risk-averse and the demand curve is downward-sloping, i.e., $d = 1$, while sometimes they are much less risk-averse and almost risk-neutral, i.e., $d = 0$. Intuitively, this is equivalent to the market depth being deterministic (i.e., not random), but speculators face noise-trading risk. In one state, noise traders are few, d is a positive number (normalized to be 1) reflecting the true market depth, and hence speculators can influence the stock price. In the other state, noise traders are abundant. They *absorb* the selling of speculators, and $d = 0$. Further, as will be shown later, in equilibrium, s is deterministic. At this stage, we assume that s is public information and $s = 2$. We further assume the following parameter values: $\theta = 7$, $K = 6$, and $\sigma = 0$.

We compute the debt value in a rational-expectations equilibrium. Note that $P = \theta - ds$. If there is no short selling, then $P = \theta = 7$. Hence, creditors can perfectly infer the debt value from the stock price: $D = min(\theta, K)$, which is 6. Suppose there is short selling. Given the fundamental value $\theta = 7$, the price P can be 7 or 5, depending on whether the market is liquid enough. In the case of $P = 7$, the creditors rationally expect (with Bayesian inference) that θ can be either 9 or 7 with equal probabilities, that is, $E(\theta|P = 7) = 8$.[16] In the case of $P = 5$, the creditors rationally expect that θ can be either 7 or 5 with equal probabilities, that is, $E(\theta|P = 5) = 6$. Therefore, the estimation of θ is unbiased, i.e., $E[E(\theta|P)|\theta] = \theta$. For the debt value, however, we can easily obtain $E(D|P = 7) = 6$ and $E(D|P = 5) = 5.5$. Hence, $E[E(D|P)|\theta] < K$. That is, short selling creates uncertainty and increases information asymmetry, causing a reduction in the expected debt value.

Graph 1 demonstrates the example.



[16] Note that θ has an 'improper uniform prior' over the real line.

Graph 1: A simple example

We have three observations from the above example. First, the more aggressive the short selling (i.e., *the higher the* s), the higher the uncertainty creditors face in inferring the true fundamental value. This is because short selling amplifies the market liquidity shocks. Second, short selling does decrease the (expected) stock price. However, the lower stock price itself is not the underlying cause for the reduction in debt value. In fact, creditors can rationally expect that a low stock price may be due to short-selling. Thus, when estimating the fundamental value θ, they would take short-selling pressure into consideration and *offset* the possible price derivation. That is, creditors can correctly estimate the fundamental θ, on average. Instead, the key reason for the reduction in the (expected) debt value is the increase in uncertainty. Third, it seems that long-side attacks can also increase uncertainty and cause the failure of the bank. However, speculators need to gain from their positions. Because only 'short' positions can benefit from the fall in the stock price (i.e., the bank failure), speculators choose to attack by shorting.

2.3 Equilibrium of the model

We assume that d has the normal distribution $d \sim N(0, h^2)$. Note that d is typically positive. The assumption of normal distribution, however, is for tractability. The mechanism of the model does not depend on this assumption.

In fact, we can assume $d \sim N(a, h^2)$, where $a > 0$. In this case, the price P at T_1 follows the distribution $P \sim N(\theta - as, h^2 s^2)$. So short selling not only depresses the mean of stock prices but also increases the variance. However, the danger of short-selling does not lie in the change in the mean rather than in the variance. That is, in equilibrium, creditors can rationally anticipate the (expected) magnitude of short-selling and hence offset the term $-as$. Using the language of econometrics, from P there is always an *unbiased* estimation of θ, but the *efficiency* of the estimation decreases. In order to highlight the fact that it is the change in variance rather than in mean that causes problems, and to have a clean analysis, we assume $a = 0$.

With the above setup, the price P at T_1 has the distribution $P \sim N(\theta, h^2 s^2)$. We also assume that $0 \ll \frac{\gamma}{h} \ll +\infty$, that is, both private and public information is valuable.

We consider the threshold (monotone) equilibrium of the model.[17] That is, both the speculators

[17] In the finance literature on applications of global games, the threshold equilibrium is the primary interest. For

and the creditors use threshold strategies. Specifically, the speculators' strategy is

$$\theta^i \longmapsto \begin{cases} Not\ short & \theta^i \geq \theta^* \\ Short & \theta^i < \theta^* \end{cases},$$

where θ^* is the threshold. The creditors use the strategy

$$(\theta^j, P) \longmapsto \begin{cases} Roll\ over & \theta^j \geq \theta^{**}(P) \\ Not\ Roll\ over & \theta^j < \theta^{**}(P) \end{cases},$$

where $\theta^{**}(P)$ is the threshold, which itself is a function of P.

Solving the equilibrium of the model means working out the two thresholds θ^* and $\theta^{**}(P)$, as shown in Figure 1. We solve the equilibrium in the following three cases to gradually expose the double-run game. First, we study one run – the creditor run – and abstract from the speculator run by assuming s as being exogenously given. Then, we move to double runs and endogenize s. Finally, we further endogenize the payoffs to short selling, t and r.

2.3.1 One run – the creditor run under noisy stock prices with exogenous s

In this subsection, we study the second stage of the double-run game – the creditor run. Our aim in this subsection is to show one key insight of the paper: creditor runs are more easily triggered when stock prices are more noisy. To focus on this analysis, we abstract from the speculative attack and assume that s is exogenously given. Recall that s represents the aggregate short-selling by the speculators at T_0.

As P has the distribution $P \sim N(\theta, h^2 s^2)$, it is clear that the higher the s, the more noisy the price. Thus, s is a measure of the degree of noise in the price. In this subsection, we abstract from the economic source of s (i.e., the reason for noise in the price). Instead, we focus on studying the effect of the noise on the trigger level for bank runs, that is, how s impacts $\theta^{**}(P)$. Global game methods enable us to explicitly express the relation.

We work out the creditors' optimal strategy at T_1 for a given s. If a creditor declines to roll over at T_1, her payoff is F. If the creditor rolls over, her (expected) payoff is the debt value

example, Morris and Shin (2004, 2009) and He and Xiong (2012) consider only threshold equilibria. As in Goldstein and Pauzner (2005), if an upper dominance region is assumed, the "bad equilibrium" is ruled out for sufficiently strong bank fundamentals. See Angeletos, Hellwig and Pavan (2006, 2007) on equilibrium multiplicity.

$D(\theta) = E[\min(\widetilde{\theta}, K)|\theta]$ conditional on the realization of θ.[18] Clearly, $D(\theta)$ is concave with respect to θ. A creditor infers the debt value based on the stock price information and her private signal. At T_1, the price P is distributed as $P \sim N(\theta, h^2 s^2)$ while the private signal has the distribution $\theta^j \sim N(\theta, \gamma^2)$. Thus, the conditional expectation of the debt value for a creditor is $E[D(\theta)|P, \theta^j; s]$.

We have already defined the threshold $\theta^{**}(P)$, which is the threshold below which the creditors decline to roll over at T_1. Here we define another threshold $\widehat{\theta}(P)$, which is the threshold of fundamental value θ below which the bank fails (for a given P) at T_1. Intuitively, θ^{**} is the 'running point' (of the creditors) while $\widehat{\theta}$ is the 'failure point' (of the bank). Solving the creditor run equilibrium means working out these two thresholds. Fixing P, we solve θ^{**} and $\widehat{\theta}$ simultaneously.

First, given θ^{**}, we work out the bank failure threshold $\widehat{\theta}$. For a given θ^{**}, the proportion of the creditors declining to roll over at T_1 conditional on the realized fundamentals θ is $\text{pr}(\theta^j < \theta^{**}|\theta) = \Phi(\frac{\theta^{**}-\theta}{\gamma})$, where $\Phi(\cdot)$ is c.d.f. of the standard normal distribution. Recall that the bank fails at T_1 if more than $\frac{L}{F}$ proportion of the creditors decline to roll over. Therefore, we have the following equation:

$$\Phi(\frac{\theta^{**} - \widehat{\theta}}{\gamma}) = \frac{L}{F}. \tag{1}$$

Next, given $\widehat{\theta}$, we consider the position for an individual creditor. In equilibrium, the creditor at margin who just receives signal θ^{**} should be indifferent about rolling over or not. That is,

$$\int_{-\infty}^{\widehat{\theta}} 0 \cdot d\theta + \int_{\widehat{\theta}}^{+\infty} D(\theta) \cdot f(\theta|P, \theta^j = \theta^{**}; s) d\theta = F, \tag{2}$$

where $f(\theta|P, \theta^j; s)$ is the posterior p.d.f. for $P \sim N(\theta, h^2 s^2)$ and $\theta^j \sim N(\theta, \gamma^2)$. The left-hand side of (2) expresses the payoff for the marginal creditor when she decides to roll over: If $\theta < \widehat{\theta}$, the bank cannot survive to T_2 and she gets nothing, which is the first term; conditional on the bank surviving to T_2, her expected payoff is $E[D(\theta)|P, \theta^j; s]$, the second term.

By solving the system of equations (1)-(2), we can obtain the pair of θ^{**} and $\widehat{\theta}$, both of which are functions of P. The two thresholds $\theta^{**}(P)$ and $\widehat{\theta}(P)$ fully characterize the equilibrium of the creditor-run game. The equilibrium threshold, $\theta^{**}(P)$, depends on s. This is because s determines the informativeness of the stock price. Intuitively, when s increases, the posterior distribution $\theta|P, \theta^j$ gets fatter tails, meaning that both downside risk and upside risk of θ increase. But the

[18] As $\widetilde{\theta} \sim N(\theta, \sigma^2)$, when σ is low, the probability of $\widetilde{\theta}$ being negative is negligible conditional on a relevant positive θ.

creditors mainly care about the downside risk because $D(\theta)$ is concave. So they may set a higher threshold $\theta^{**}(P)$. Also, $\theta^{**}(P)$ depends on L, which measures the degree of maturity mismatch.

We need to examine the existence and uniqueness of the threshold equilibrium. By (1), we have $\widehat{\theta} = \theta^{**} - \gamma \cdot \Phi^{-1}(\frac{L}{F})$. Thus, we can combine (1) and (2) as

$$\int_{\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{+\infty} D(\theta) \cdot f(\theta | P, \theta^j = \theta^{**}; s) d\theta = F. \tag{3}$$

If the private signal is infinitely more precise than the public signal, it is easy to prove that (3) admits a unique solution of $\theta^{**}(P)$. In fact, in the limit $\gamma \to 0$ for given h, (3) can be transformed to:

$$\frac{L}{F} \cdot D(\theta^{**}) = F.$$

The above equation becomes very intuitive: the term $\frac{L}{F}$ measures strategic (coordination) risk while $D(\theta^{**})$ corresponds to fundamental risk. The uniqueness of the solution of θ^{**} is straightforward (under some proper parametric values). Also, the equation clearly shows that θ^{**} is decreasing in L. The intuition behind the derivation of the above equation is as follows. Under the limit $\gamma \to 0$, fundamental uncertainty disappears, i.e., $\theta \to \theta^{**}$. However, strategic uncertainty does not. From the perspective of the marginal creditor, whose signal is θ^{**}, the proportion of the creditors calling loans is uniformly distributed within $[0, 1]$. So the probability for the proportion of the creditors calling loans being less than $\frac{L}{F}$ is exactly $\frac{L}{F}$ in the eyes of the marginal creditor.

In our setting, we stipulate that the public information is not infinitely small (i.e., $\frac{\gamma}{h} \gg 0$). We show that (3) admits a unique stable equilibrium for some values of $\frac{\gamma}{h}$. We focus on the stable equilibrium based on the "equilibrium selection in global games with strategic complementarities" in Frankel, Morris and Pauzner (2003) and Vives (2005, 2013).[19]

Proposition 1 summarizes the equilibrium and the comparative statics.

Proposition 1 *When $\frac{\gamma}{h}$ is not too high or too low, the creditor-run game has a unique (stable) threshold equilibrium, and the equilibrium threshold, $\theta^{**}(P)$, increases in s and decreases in L, that is, $\frac{\partial\theta^{**}(P)}{\partial s} \geq 0$ and $\frac{\partial\theta^{**}(P)}{\partial L} \leq 0$.*[20]

[19] In a (local) "stable" equilibrium, the best response function (of an individual player to its peers) intersects the 45^0 line at a slope of less than 1. See also Kajii and Morris (1997) and Morris and Shin (2003).

[20] Similar to other global games models, there are conditions on $\frac{\gamma}{h}$. In our paper, $\frac{\gamma}{h}$ cannot be too low because we want to make sure that the public information has impacts and to guarantee the comparative statics.

Proof: see Appendix B.

The intuition behind the comparative statics in Proposition 1 is as follows. The credit risk of a financial institution can be decomposed into two parts: *insolvency risk* and *illiquidity risk*. In our model, an increase in s leads to a greater uncertainty on the bank asset value, which implies higher insolvency risk. A lower L means a more severe coordination problem among the creditors, which causes higher illiquidity risk. Both sources of risk lead to creditors' running at a higher threshold $\theta^{**}(P)$.

Proposition 1 explicitly expresses the relation between the degree of noise in stock prices and the trigger level for bank runs. The higher the degree of noise, the higher the trigger level. In the next subsection, we analyze the role of speculative attacks in creating the noise.

2.3.2 Double runs with endogenous s

In this subsection, we endogenize s. In doing so we are able to know the exact degree of noise in the stock price in equilibrium. We show that the atomistic speculators *automatically* coordinate to attack, increasing noise in stock prices and thereby triggering a creditor run at a higher level. That is, we show that there exist *endogenous* strategic complementarities among speculators.

We have two global games played subsequently and interacting in equilibrium. We work out the equilibrium by backward induction, from T_1 to T_0. We find the equilibrium in three steps:

Step 1: Given the speculators' strategy at T_0, we work out the creditors' optimal strategy at T_1;

Step 2: Given the strategy used by the creditors in Step 1, we calculate the bank failure probability at T_1;

Step 3: We go back to T_0 and work out the equilibrium strategy among the speculators.

Proposition 2 summarizes the equilibrium of the double-run game, which contains two intertwined global games.

Proposition 2 *Under some parametric values, there exists a unique (stable) threshold equilibrium for the double-run game, characterized by the pair (θ^*,$\theta^{**}(P)$).*

We provide the proof of Proposition 2 after discussing the implications. With the equilibrium result, we are able to discuss the key insight of the model in this subsection: the speculator run and the creditor run not only interact but also reinforce each other in equilibrium. Formally, we have $\frac{\partial\theta^*}{\partial\theta^{**}(P)} > 0$ and $\frac{\partial\theta^{**}(P)}{\partial\theta^*} > 0$. Intuitively, $\theta^{**}(P)$ is the 'running point' (of the creditors) while θ^* is the 'attacking point' (of the speculators). The two points reinforce each other in equilibrium. If $\theta^{**}(P)$ increases (i.e., the creditors run on the bank at a higher level), then it is easy to bring down the bank, which induces the speculators to start shorting at a higher threshold θ^*; this is $\frac{\partial\theta^*}{\partial\theta^{**}(P)} > 0$. Conversely, if the speculators are more aggressive in shorting (i.e., a higher θ^*), creditors understand that the stock price is noisier and thus run at a higher threshold, which is $\frac{\partial\theta^{**}(P)}{\partial\theta^*} > 0$.

In equilibrium, θ^* and $\theta^{**}(P)$ are endogenous while t and L are exogenous. Note that t directly impacts θ^* while L directly impacts $\theta^{**}(P)$. It is easy to obtain the following important comparative statics.

Proposition 3 *When $\frac{\gamma}{h}$ is not too high or too low, the speculator run and the creditor run reinforce each other in equilibrium in that $\frac{\partial\theta^*}{\partial\theta^{**}(P)} > 0$ and $\frac{\partial\theta^{**}(P)}{\partial\theta^*} > 0$. Therefore, given a negative shock on L or a positive shock on t, the equilibrium thresholds θ^* and $\theta^{**}(P)$ spiral upward.*

Proof: see Appendix B.

The comparative static results in Proposition 3 are important because they formally show that a small change in the exogenous variables can significantly change the equilibrium thresholds due to the two-way feedback. Figures 2a and 2b illustrate the upward spirals, where $\theta^*(\theta^{**})$ and $\theta^{**}(\theta^*)$ are the reaction functions between θ^* and $\theta^{**}(P)$ for given realized Ps. Look at the effect of a small change in L in Figure 2a, where the feedback loop is $\frac{\partial\theta^{**}}{\partial L} < 0$, $\frac{\partial\theta^*}{\partial\theta^{**}} > 0$, $\frac{\partial\theta^{**}}{\partial\theta^*} > 0$. The intuition is as follows. A decrease in L leads to a more severe coordination problem among creditors, resulting in the creditors' running at a higher threshold (that is, $\frac{\partial\theta^{**}}{\partial L} < 0$); consequently, it is easy to bring down the bank, and that in turn induces the speculators to start to short at a higher threshold (that is, $\frac{\partial\theta^*}{\partial\theta^{**}} > 0$). The effect, however, does not stop here. The more aggressive short selling by the speculators exacerbates the fundamental uncertainty, which makes a creditor run even more likely (that is, $\frac{\partial\theta^{**}}{\partial\theta^*} > 0$), leading to the speculators' shorting at an even higher threshold, and so on in an upward spiral. The effect of a change in t is similar, where the feedback loop is $\frac{\partial\theta^*}{\partial t} > 0$, $\frac{\partial\theta^{**}}{\partial\theta^*} > 0$, $\frac{\partial\theta^*}{\partial\theta^{**}} > 0$.

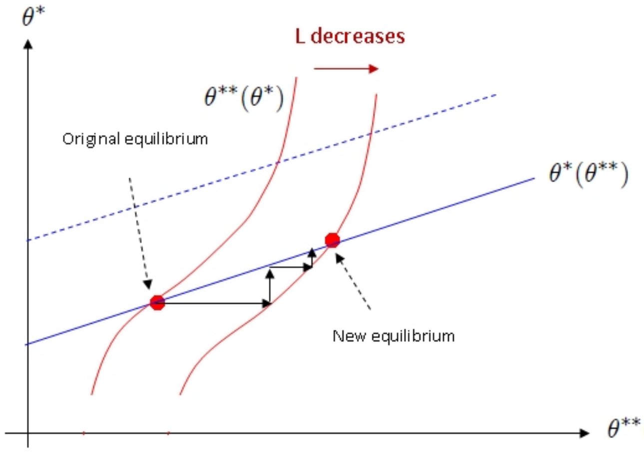


Figure 2a: The equilibrium under constant payoffs to short selling

The feedback loop for a negative shock on L:

$\frac{\partial\theta^{**}}{\partial L} < 0$, $\frac{\partial\theta^{*}}{\partial\theta^{**}} > 0$, $\frac{\partial\theta^{**}}{\partial\theta^{*}} > 0$

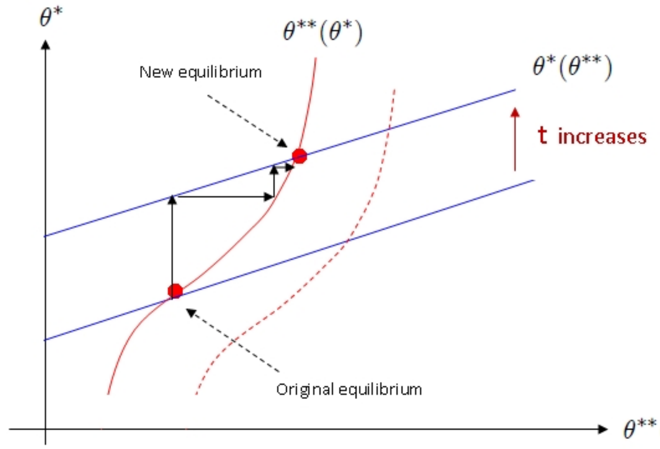


Figure 2b: The equilibrium under constant payoffs to short selling

The feedback loop for a positive shock on t:

$\frac{\partial\theta^{*}}{\partial t} > 0$, $\frac{\partial\theta^{**}}{\partial\theta^{*}} > 0$, $\frac{\partial\theta^{*}}{\partial\theta^{**}} > 0$

Now we proceed to the proof of Proposition 2 in three steps with a series of lemmas.

Step 1 Conditional on the speculators using the threshold θ^* and the fundamental value being θ, the proportion of the speculators short selling at T_0 is $s(\theta;\theta^*) = \Pr(\theta^i < \theta^*|\theta) = \Phi(\frac{\theta^*-\theta}{\delta})$. So $s(\theta;\theta^*)$ is increasing in θ^* and decreasing in θ.

We have solved the equilibrium for the creditor-run game in the last subsection. Here we only need to substitute the endogenous $s(\theta;\theta^*)$ for the exogenous s in equation (3). That is,

$$\int_{\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{+\infty} D(\theta)\cdot f(\theta|P,\theta^j=\theta^{**};\theta^*)d\theta = F, \tag{4}$$

where $f(\theta|P,\theta^j;\theta^*)$ is the posterior p.d.f. for $P \sim N(\theta,(h\cdot s(\theta;\theta^*))^2)$ and $\theta^j \sim N(\theta,\gamma^2)$.

As in Proposition 1, we have the comparative statics on the creditor-run game. Lemma 1 follows.

Lemma 1 *Under some parametric values, the creditor-run game has a unique (stable) threshold equilibrium, and the equilibrium threshold, $\theta^{**}(P)$, increases in θ^* and decreases in L, that is, $\frac{\partial\theta^{**}(P)}{\partial\theta^*} \geq 0$ and $\frac{\partial\theta^{**}(P)}{\partial L} \leq 0$.*

Proof: See Appendix B.

Step 2 We can work out the probability of bank failure, which ultimately depends on θ, θ^* and L. Denote the bank failure probability by $\Pr(\theta;\theta^*,L)$, where $\Pr(\theta;\theta^*,L) = \Phi(\frac{\widehat{\theta}^{-1}(\theta)-\theta}{h\cdot s(\theta;\theta^*)})$.[21] We

[21]The complete results of the probability of bank failure are provided in the appendix.

have the following property regarding $\Pr(\theta;\theta^*,L)$.

Lemma 2 *The bank failure probability* $\Pr(\theta;\theta^*,L)$ *decreases in* θ*. For a given* θ*,* $\Pr(\theta;\theta^*,L)$ *increases in* θ^* *when* θ *is high enough, and decreases in* L *.*

Proof: see Appendix B.

The intuition behind Lemma 2 is simple. The higher the fundamental value θ, the lower the bank failure probability. The more aggressive the short-selling attack (i.e., the higher θ^* and thus the higher threshold $\widehat{\theta}(P)$), the higher the bank failure probability. Finally, the higher the degree of maturity mismatch (i.e., the lower L and thus the higher $\widehat{\theta}(P)$), the higher the bank failure probability.

Step 3 A speculator's action depends on her belief about other speculators' actions. Conditional on all other speculators using the threshold strategy with the threshold being θ^* and *creditors using the strategy in Step 1 as the response to speculators*, we compute the optimal strategy of speculator i. The threshold of speculator i, denoted by θ^{i*}, satisfies:

$$\int_{-\infty}^{+\infty} \{[\Pr(\theta;\theta^*,L)\cdot t + (1-\Pr(\theta;\theta^*,L))\cdot r] - c\}\cdot dG(\theta|\theta^{i*}) = 0, \tag{5}$$

where $G(\theta|\theta^i)$ is the posterior c.d.f. for $\theta^i \sim N(\theta,\delta^2)$. The left hand side of (5) is the expected net payoff if speculator i decides to attack while the right hand side is her payoff if she does not attack. By the definition of θ^{i*}, she is indifferent about attacking or not when she receives signal θ^{i*}.

We show that, given θ^*, there exists a unique solution of θ^{i*} in (5). Further, if $\frac{c-r}{t-r}$ is small enough (e.g., $\frac{c-r}{t-r} \leq \frac{1}{2}$) or $\frac{F}{K}$ is high enough, θ^{i*} is increasing in θ^*.

Lemma 3 θ^{i*} *is increasing in* θ^**. That is, there exist strategic complementarities among the speculators in short selling.*

Proof: See Appendix B.

Lemma 3 is an important result. It formally shows that there exist strategic complementarities among short-sellers: If other speculators are aggressive in short selling, it is optimal for a particular individual speculator to be aggressive as well. Importantly, the strategic complementarities among short-sellers in our paper are *endogenous*, which differs from Morris and Shin (1998). It is through the impact on creditors' rollover decisions that the speculators create strategic complementarities among themselves.

After showing the strategic complementarities, we are able to show that there exists a unique threshold equilibrium among the speculators. By symmetric equilibrium, we have

$$\theta^{i*} = \theta^*. \tag{6}$$

We can combine equations (5) and (6) to obtain one equation:

$$\int_{-\infty}^{+\infty} \{[\Pr(\theta;\theta^*,L)\cdot t + (1-\Pr(\theta;\theta^*,L))\cdot r] - c\} \cdot dG(\theta|\theta^*) = 0. \tag{7}$$

By solving (7), we obtain θ^*. We have comparative statics: θ^* is an increasing function of t. That is, the higher the gain of short selling, the more aggressive the short-sellers are. Lemma 4 follows.

Lemma 4 *Under some parametric values, there exists a unique threshold equilibrium among speculators in the short-selling attack, given by equation (7). The equilibrium threshold θ^* is an increasing function of t, that is, $\frac{\partial\theta^*}{\partial t} > 0$.*

Proof: See Appendix B.

To summarize, equations (4) and (7) together give the equilibrium of the double-run game. Equation (4) expresses the creditor run while equation (7) characterizes the speculator run. The two runs interact with each other.

2.3.3 Double runs with endogenous s and (t, r)

So far we assume that the payoffs to short selling are constant. In this subsection, we endogenize the payoffs to short selling, highlighting another layer of amplifying mechanism.

In our model, the Walrasian auctioneer sets the stock price given by $p = v - ds$. If the bank fails, the speculators spend zero money to buy the stock back; the payoff (gain) of short selling is $v - ds$. In contrast, if the bank does not fail, the speculators' payoff (loss) is $-ds$. Conditional on the fundamental value θ, the expected gain is $t = E(v - ds|\theta) = e(\theta) = \theta - K$ and the expected loss is $r = E(-ds|\theta) = 0$, considering that $d \sim N(0, h^2)$.[22] Therefore, under the endogenous payoffs to

[22] Under the alternative assumption $d \sim N(a, h^2)$, we have that $t = E(v - ds|\theta) = e(\theta) - as(\theta;\theta^*)$ and $r = E(-ds|\theta) = -as(\theta;\theta^*)$. Considering that $s(\theta;\theta^*)$ is a decreasing function of θ, when θ increases, the gain t increases while the loss $|r|$ decreases. So, under the alternative assumption, the model result does not change if the sufficient condition that a is small enough is satisfied. Also, we can substitute the equity value with $e(\theta) = E[\max(0, \widetilde{\theta} - K)|\theta]$, rather than its reduced form $e(\theta) = \theta - K$. As long as $e(\theta)$ is increasing in θ, the insight of the model is intact.

short selling, equation (7) is replaced by the system of equations (8a)-(8b):

$$\int_{-\infty}^{+\infty} [\Pr(\theta; \theta^*, L) \cdot t] \cdot dG(\theta|\theta^*) = c \tag{8a}$$

$$t = \theta - K \tag{8b}$$

By comparing (8a)-(8b) with (7), we can see positive feedback arising under the endogenous payoff scheme: the higher the gain t of short selling, the higher the threshold θ^* by Lemma 4; a higher threshold θ^* drives gain $t = \theta - K$ further higher (in the stochastic sense) because $E(\theta - K|\theta^*) = \theta^* - K$, and so on in a spiral. Formally, we have that

$$\frac{\partial E(t|\theta^*)}{\partial \theta^*} > 0 \quad \text{in (8b)} \qquad \text{and} \qquad \frac{\partial \theta^*}{\partial t} > 0 \quad \text{in (8a).}$$

Therefore, for the double-run game with the endogenous payoffs to short selling, a decrease in L raises the levels of θ^* and $\theta^{**}(P)$ along the compound spirals in that $\frac{\partial \theta^{**}(P)}{\partial L} < 0$, $\frac{\partial \theta^*}{\partial \theta^{**}(P)} > 0$, $\frac{\partial \theta^{**}(P)}{\partial \theta^*} > 0$, $\frac{\partial E(t|\theta^*)}{\partial \theta^*} > 0$, $\frac{\partial \theta^*}{\partial t} > 0$. Figure 3 illustrates the idea.

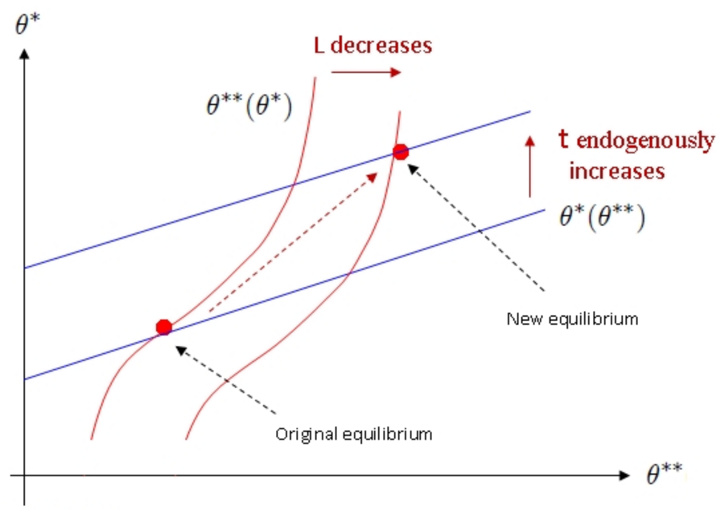


Figure 3: The equilibrium under endogenous payoffs to short selling

The feedback loop for a negative shock on L:

$$\frac{\partial \theta^{**}}{\partial L} < 0,\ \frac{\partial \theta^*}{\partial \theta^{**}} > 0,\ \frac{\partial \theta^{**}}{\partial \theta^*} > 0,\ \frac{\partial E(t|\theta^*)}{\partial \theta^*} > 0,\ \frac{\partial \theta^*}{\partial t} > 0$$

Intuitively, in our model, the creditors' running threshold affects the aggressiveness of the speculators through two channels. A high running threshold not only makes the short-selling attack succeed more easily but also increases the gain of short selling in case of a successful attack. These two forces jointly push up the equilibrium threshold θ^* (that is, $\frac{\partial \theta^*}{\partial \theta^{**}(P)} > 0$, $\frac{\partial E(t|\theta^*)}{\partial \theta^*} > 0$, $\frac{\partial \theta^*}{\partial t} > 0$).

3 Predictions and implications of the model

3.1 Predictions of the model

In the model, the short-selling attack probability is $s(\theta; \theta^*)$ while the bank failure probability is $\Pr(\theta; \theta^*, L)$. In equilibrium, θ^* is determined, a function of L, shown in Propositions 2 and 3 as well as in Figure 3. Therefore, the two probabilities ultimately depend on only two parameters: the level of fundamentals, θ, and the degree of maturity mismatch, L; let $\text{Prob}(\theta; L)$ denote the bank failure probability under endogenous θ^*. We can easily conduct the comparative static analysis on these and derive empirical predictions. We have two cross-sectional predictions of the model.

Prediction 1: Banks with lower fundamentals are more likely to incur short-selling attacks and fail with a higher probability.

Prediction 1 shows the link between the panic-driven run and the fundamentals-driven run. In our model, speculation can only exacerbate the problem but is not the origin of the problem.

Prediction 2: The more severe the maturity (liquidity) mismatch is, the more likely the short-selling attack is and the higher the probability that the firm fails.

Prediction 2 explains why banks rather than standard corporations are more likely to incur short-selling attacks. In our model, the maturity (liquidity) mismatch of balance sheets is a key factor driving the probability of short-selling attacks and the likelihood of firm failure. Banks are more likely to incur, and are more vulnerable to, short-selling attacks because banks typically have a much higher degree of maturity (liquidity) mismatch than standard corporations.

3.2 Policy implications

We discuss policy implications of the model from the ex post and ex ante perspectives.

Ex post, short selling destroys value and is socially inefficient. Our model shows that short selling can fuel a crisis. Short selling can cause the failure of a viable bank. Banks with weak fundamentals and severe maturity mismatches are most vulnerable to short-selling attacks. Those banks are the first to fall. The model highlights that banks can be subject to 'double runs'. Also, short selling can potentially create systemic risk. In our model, if the fall of one bank or some banks imposes negative externality to other banks by reducing the fundamentals of other banks, then even healthy banks might become weaker. This would trigger a new round of short-selling attacks. This way short selling can lead to a chain of collapses of financial institutions.

In order to make clear how short selling exacerbates stability, we compare three cases.

Case 1: Short-selling ban In our paper's setup, we have $P = \theta - ds$, so if short selling is banned: $s = 0$, then $P = \theta$. Hence, without short selling, the stock price perfectly reflects the long-term fundamentals of the bank. In this case, the bank fails if and only if its fundamental value is below threshold θ^T, where θ^T solves $D(\theta^T) = F$.[23] So the bank fails with probability 1 when $\theta < \theta^T$ and with probability 0 otherwise.

Case 2: Allowing short selling but no coordination problems among creditors We can think of this case as about short selling on a financial firm with one single large lender (or on a non-financial firm), where there is no coordination problem among creditors. So the creditors' withdrawal threshold $\theta^{**}(P)$ satisfies $E[D(\theta)|P, \theta^j = \theta^{**}] = F$. We can calculate the bank's failure probability in equilibrium as we do in Lemma 2. In comparison with Case 1, short selling results in two inefficiencies: 1) Noise-creating ("accident") effect: the bank with a realization of good fundamentals can fail and with bad fundamentals can survive under the *noisy* stock price as public information; 2) Uncertainty-increasing effect: because of creditors' concave payoffs, uncertainty makes creditors withdrawing at a higher threshold; so the bank failure probability increases, on average, compared with Case 1.

Case 3: Allowing short selling and with coordination problems among creditors In this case, creditors need to be concerned about not only the insolvency risk of the bank but also its interim illiquidity (coordination) risk. Intuitively (and roughly) speaking, the creditors' withdrawal threshold in this case is the threshold in Case 2 scaled up by $\frac{F}{L}$, recalling that $\frac{F}{L}$ measures the illiquidity (coordination) risk of the bank as shown in (3). So the bank's failure probability is increased. Because of the compound feedback loops between the first stage and the second stage in our model, in equilibrium the creditors' withdrawal threshold should be scaled up by greater than $\frac{F}{L}$, so the bank's failure probability can be much higher.[24]

Figure 4 depicts the bank failure probability under the three cases.

[23] The social optimal liquidation threshold is $\theta = L$.

[24] The stock prices are also noisier in case 3 than in case 2. So the probability of the "accident" that a bad bank ends up with a high stock price and thus can survive also increases.

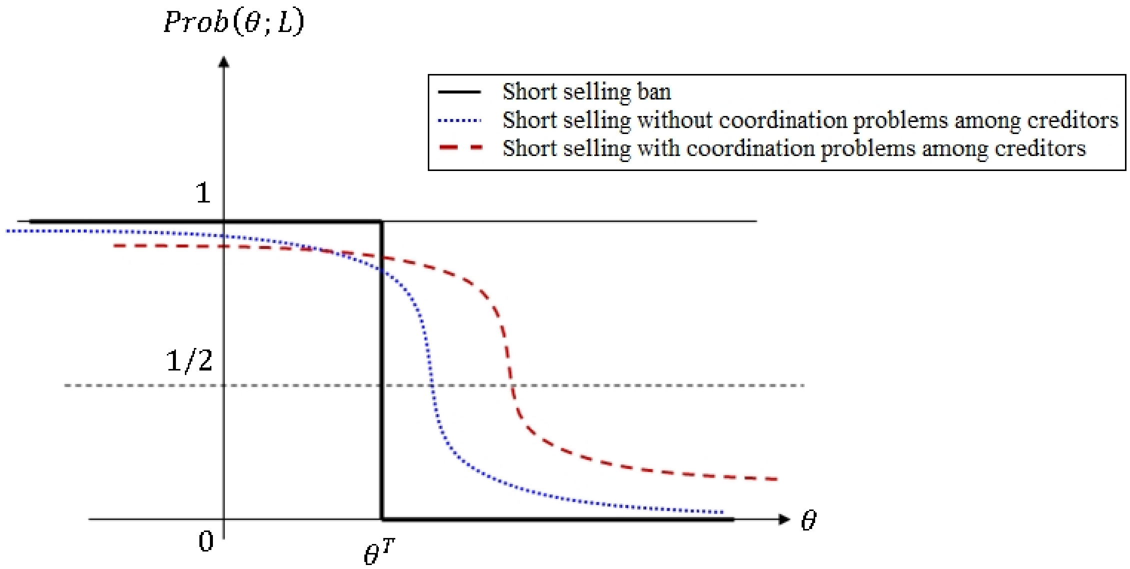


Figure 4: Bank failure probability under three cases

From the ex ante perspective, however, the ban on short selling has side effects. Suppose in our model that the fundamental value θ and the liquidity status L are partially determined by the bank manager's effort ex ante (before T_0). The greater the effort, the higher the probability that the bank realizes a high fundamental value θ. Hence, if there exist short-selling threats, the bank manager has incentives to work harder ex ante. In fact, in our model, short-selling increases the bank failure probability *disproportionally* for different fundamentals. For a high fundamental value, the bank is little affected by short selling. In contrast, if the fundamental value is in the intermediate region, short selling drastically increases the failure probability. Consequently, with short-selling threats, the bank manager has incentives to work hard ex ante, trying to realize a high fundamental value and thus to minimize the risk of being exposed to a high chance of failure. Also, L is a measure of how liquid a bank's asset is. Hence, without short-selling threats, banks can be induced to over-invest in illiquid assets.

4 Concluding remarks

The paper provides a theoretical model for understanding how short selling could threaten the fair and orderly functioning of financial markets. Note that unlimited shorting is made possible by "naked" trading. We argue that short selling can sometimes create uncertainty and increase information asymmetry. We highlight that speculator runs and creditor runs can co-exist and in fact mutually reinforce each other.

In our model, bank runs are triggered by an increase in uncertainty on fundamentals. This links the panic-based run (as in Bryant (1980)) and the fundamental-based run (as in Allen and Gale (1998)). In fact, in our model, on one hand, creditors receive the fundamental-surrounding (i.e., payoff-related) information and thus the equilibrium is not a sunspot equilibrium (i.e., pure panic), and, on the other hand, the information is imperfect and dispersed and thus there exists a certain degree of coordination problems among creditors. Our paper highlights that the coordination problem among creditors can be amplified by an increase in uncertainty on fundamentals through short-selling attacks. A bank that has fundamentals at margin (i.e., viable but weak) is most affected by short-selling attacks.

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6 Appendix A: A numerical example

We report a simulation exercise under a set of parameter values to illustrate the results in every step of the model. We choose parameter values as: $K = 8$, $\sigma = 6$, $\delta = 3$, $h = 4$, $\gamma = 4$, $L = 6$, $F = 7.3$. The endogenous variables in the model are θ^*, θ^{**} and $\widehat{\theta}$.

Creditors' equilibrium strategy Figure A-1 depicts the results in Lemma 1: the creditors' equilibrium withdrawing threshold $\theta^{**}(P)$ (for different θ^*).

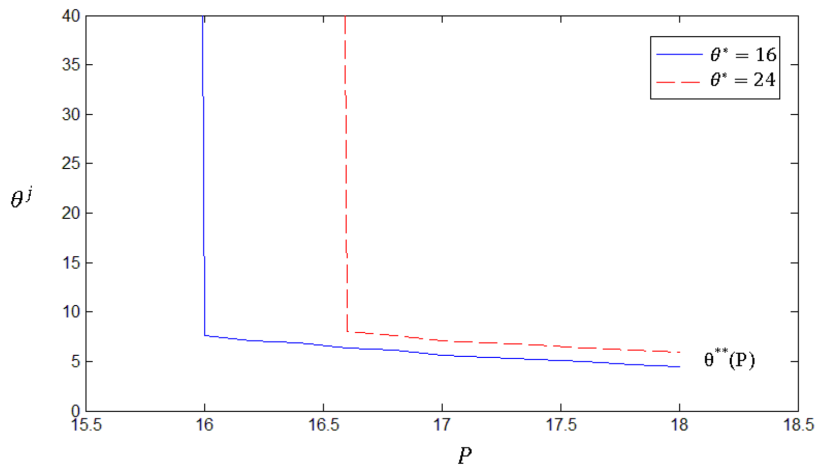

Figure A-1: A numerical example: creditors' strategy

Bank failure probability Figure A-2 shows the results in Lemma 2: the bank failure probability $\Pr(\theta; \theta^*, L)$ (for different θ^*).

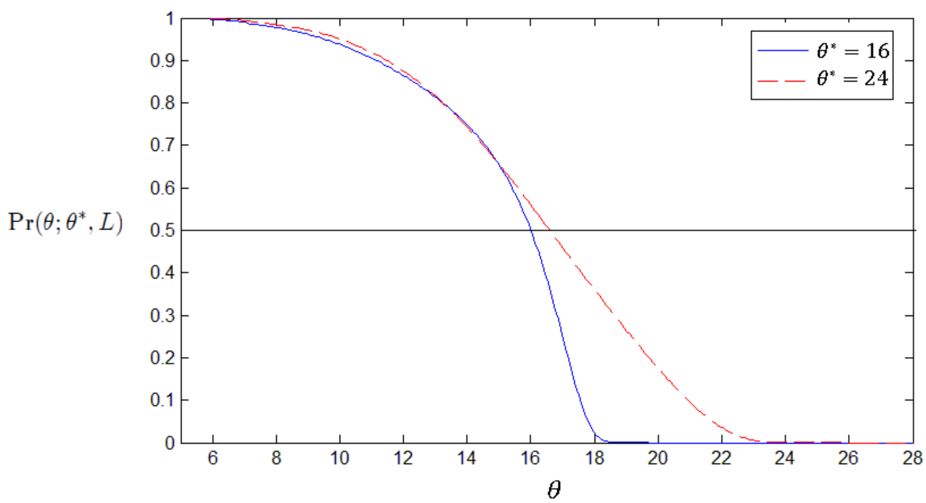

Figure A-2: A numerical example: bank failure probability

Strategic complementarities among speculators Figure A-3 demonstrates the result in Lemma 3: strategic complementarities among the speculators, where the payoff parameters of speculators satisfy $\frac{c-r}{t-r} = 0.2$.

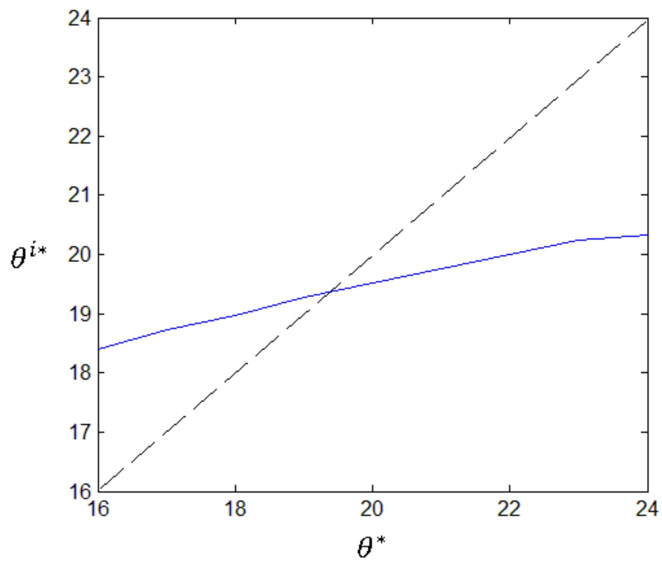

Figure A-3: A numerical example: strategic complementarities among speculators (exogenous payoffs to short selling)

Speculators' equilibrium strategy As shown in Lemma 4, the equilibrium threshold of short selling, θ^*, corresponds to the interaction between the curve $\theta^{i*}(\theta^*)$ and the 45^0 line in Figure A-3. So we have the equilibrium threshold of short selling: $\theta^* = 19.4$.

Equilibrium under endogenous payoffs to short selling Similar to Figure A-3 (Lemma 3), Figure A-4 shows strategic complementarities among the speculators under endogenous payoffs to short selling, where the parameter of the fixed cost of conducting short selling is $c = 1$. We find that the equilibrium threshold of short selling is $\theta^* = 21.6$, which corresponds to the interaction between the curve $\theta^{i*}(\theta^*)$ and the 45^0 line in Figure A-4.

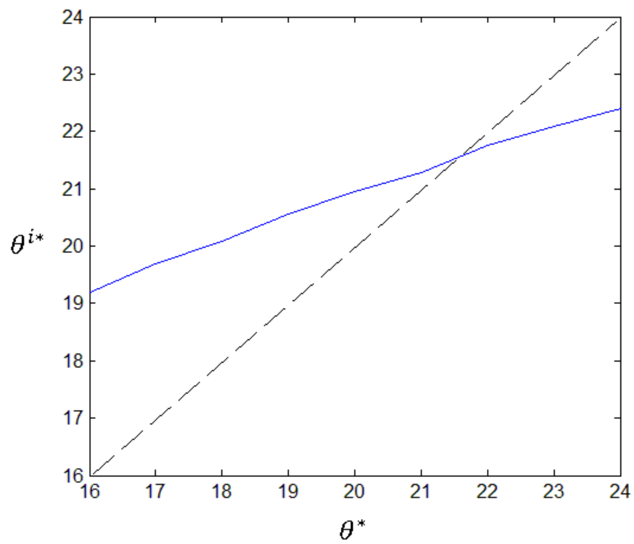

Figure A-4: A numerical example: strategic complementarities among speculators (endogenous payoffs to short selling)

Amplification effect of maturity mismatch If L decreases to $L = 5.8$ from $L = 6$, we find that the short-selling attacking threshold increases to $\theta^* = 19.7$ from $\theta^* = 19.4$ under exogenous payoffs to short selling, and increases to $\theta^* = 22.2$ from $\theta^* = 21.6$ under endogenous payoffs to short selling.

7 Appendix B: Proofs

Illustration in Section 2.1.3:

We consider $v \equiv e(\theta) = E[\max(0, \widetilde{\theta} - K)|\theta]$. Note that the equity value $e(\theta)$ is convex in θ. With the equity value being $e(\theta)$, the distribution of p becomes $p \sim N(e(\theta), (h \cdot s(\theta; \theta^*))^2)$. We show that as long as $e(\theta)$ is increasing and weakly convex in θ, the conditional expectation $E(D(\theta)|p; \theta^*)$ is decreasing in θ^*. Note that the inference route is $p \rightarrow \theta \rightarrow D(\theta)$. As long as $e(\theta)$ is weakly convex in θ, the inverse function $\theta = e^{-1}(p)$ is weakly concave. Therefore, if θ^* increases (i.e., the volatility increases), then the conditional variance $var(\theta|p)$ goes up and the conditional mean $E(\theta|p)$ goes down. Note that $D(\theta)$ is concave in θ further. Hence, through both the mean and the variance channels, the conditional expectation $E(D(\theta)|p)$ goes down if θ^* increases.

Proof of Proposition 1 and Lemma 1:

Because s is an increasing function of θ^*, proving Lemma 1 is equivalent to proving Proposition 1. So we focus on proving Lemma 1.

Considering that θ has an improper *uniform* prior over the real line (see De Groot (1970, p. 191)), given $P \sim N(\theta, (h \cdot s(\theta;\theta^*))^2)$ and $\theta^j \sim N(\theta, \gamma^2)$, where $s(\theta;\theta^*) = \Phi(\frac{\theta^*-\theta}{\delta})$, the posterior density $f(\theta|P, \theta^j; \theta^*)$ can be explicitly written as

$$\begin{aligned}
f(\theta|P,\theta^j;\theta^*) &= \frac{f(P,\theta^j|\theta;\theta^*)}{\int_{-\infty}^{+\infty} f(P,\theta^j|\theta;\theta^*)d\theta} \\
&= \frac{f(P|\theta;\theta^*)\cdot f(\theta^j|\theta)}{\int_{-\infty}^{+\infty} f(P|\theta;\theta^*)\cdot f(\theta^j|\theta)d\theta} \\
&= \frac{(\frac{1}{\sqrt{2\pi}\cdot(h\cdot\Phi(\frac{\theta^*-\theta}{\delta}))}\exp(\frac{-(P-\theta)^2}{2(h\cdot\Phi(\frac{\theta^*-\theta}{\delta}))^2}))\cdot(\frac{1}{\sqrt{2\pi}\cdot\gamma}\exp(\frac{-(\theta^j-\theta)^2}{2\gamma^2}))}{\int_{-\infty}^{+\infty}(\frac{1}{\sqrt{2\pi}\cdot(h\cdot\Phi(\frac{\theta^*-\theta}{\delta}))}\exp(\frac{-(P-\theta)^2}{2(h\cdot\Phi(\frac{\theta^*-\theta}{\delta}))^2}))\cdot(\frac{1}{\sqrt{2\pi}\cdot\gamma}\exp(\frac{-(\theta^j-\theta)^2}{2\gamma^2}))d\theta}.
\end{aligned}$$

For ease of exposition, we repeat equation (4) here:

$$\int_{\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{+\infty} D(\theta)\cdot f(\theta|P,\theta^j=\theta^{**};\theta^*)d\theta = F. \tag{A1}$$

We write LHS of (A1) as the function $U(\theta^{**}, P; \theta^*)$. If the private signal is infinitely more precise than the public signal, it is easy to prove that equation (A1) admits a unique solution of $\theta^{**}(P)$. In fact, in the limit $\gamma \to 0$ for given h, we have that

$$\begin{aligned}
&U(\theta^{**},P;\theta^*) \\
= \ &D(\theta^{**})\int_{\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{+\infty}\frac{1}{\gamma}\phi(\frac{\theta-\theta^{**}}{\gamma})d\theta \\
= \ &D(\theta^{**})\Phi(\frac{\theta^{**}-(\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F}))}{\gamma}) \\
= \ &D(\theta^{**})\frac{L}{F}.
\end{aligned}$$

In the above, $\phi(\cdot)$ stands for the p.d.f. of the standard normal. Therefore, (A1) can be rewritten as $\frac{L}{F}\cdot D(\theta^{**}) = F$. The uniqueness of the solution of θ^{**} is straightforward. Also, in the limit

$h \to +\infty$ for a given γ, for a small positive $\triangle$, we have

$$\begin{aligned} U(\theta^{**}+\triangle) &= \int\limits_{\theta=\theta^{**}+\triangle-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{\theta=+\infty} D(\theta)\cdot f(\theta|\theta^j=\theta^{**}+\triangle)d\theta \\ &= \int\limits_{\theta'=\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{\theta'=+\infty} D(\theta'+\triangle)\cdot f(\theta'|\theta^j=\theta^{**})d\theta' \\ &> U(\theta^{**}) \end{aligned}$$

In the second line above, $\theta'=\theta-\triangle$. The third line follows since $D(\theta)$ is an increasing function. Because $U(\theta^{**})$ is monotonically increasing in θ^{**}, there exists a unique solution for $U(\theta^{**})=F$.

We are interested in the case in which the private signal is not infinitely more precise than the public signal. We show that (A1) admits either zero or two solutions for some values of $\frac{\gamma}{h}$. If P is very low, (A1) has no solution. For higher values of P, (A1) admits two solutions. First, it is easy to show that $\frac{\partial U(\theta^{**},P;\theta^*)}{\partial P}>0$. Second, we show that for any given P, $U(\theta^{**},P;\theta^*)$ is a $\cap$-shape function of θ^{**}, i.e., it increases first and then decreases. In fact,

$$\frac{\partial U(\theta^{**})}{\partial\theta^{**}}=\int_{\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{+\infty} D(\theta)\cdot\frac{\partial f(\theta|P,\theta^j=\theta^{**};\theta^*)}{\partial\theta^{**}}d\theta-\left[D(\theta)\cdot f(\theta|P,\theta^j=\theta^{**};\theta^*)\right]|_{\theta=\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}.$$

An increase in θ^{**} has two effects on $U(\theta^{**},P;\theta^*)$. It not only shifts the density function $f(\theta|P,\theta^j=\theta^{**};\theta^*)$ to the right (i.e., the 'mean' of θ increases) but also pushes up the lower boundary of the integral, $\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})$. The first effect on $U(\theta^{**},P;\theta^*)$ is positive while the second effect is negative. Also, the speed of the density function moving is lower than that of the lower boundary moving, which is because the 'mean' is the weighted average of P and θ^{**} while P doesn't change. In fact, only when $h=+\infty$, are the two speeds the same. Formally, because $D(\theta)$ is increasing and concave in θ, we have that $\frac{\partial U(\theta^{**})}{\partial\theta^{**}}>0$ when θ^{**} is sufficiently small and $\frac{\partial U(\theta^{**})}{\partial\theta^{**}}<0$ when θ^{**} is sufficiently large. Also, $U(\theta^{**})$ is single-peaked in θ^{**} for some values of $\frac{\gamma}{h}$ since $\frac{\partial^2 U(\theta^{**})}{\partial\theta^{**2}}<0$ for the intermediate range of θ^{**}. Figure A-5 shows the function $U(\theta^{**},P;\theta^*)$ for a set of parameter values, where $K=8$, $\sigma=6$, $\delta=3$, $h=4$, $\gamma=4$, $L=6$, $F=7.3$, $\theta^*=18$.

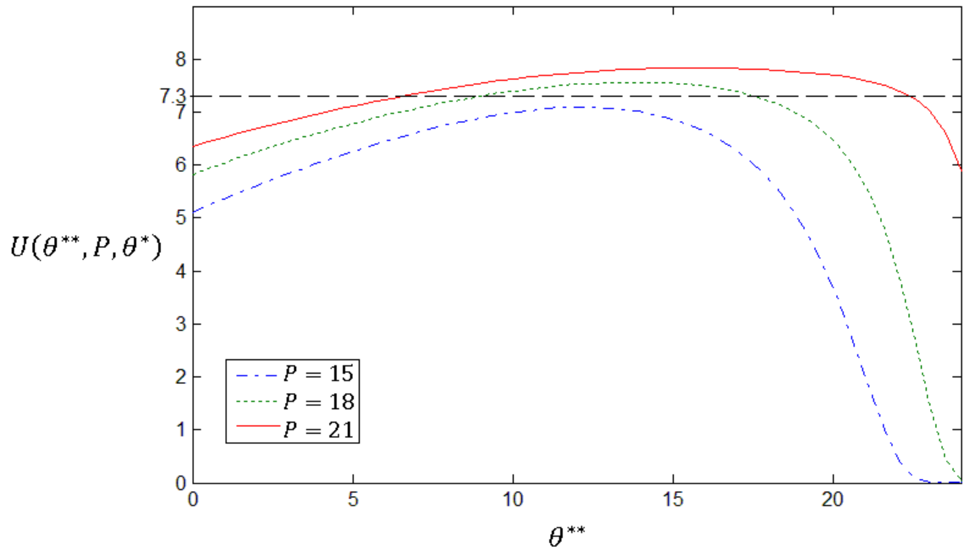

Figure A-5: The payoff function of rolling over for creditors

Next, we prove that the lower solution corresponds to a stable equilibrium while the higher solution corresponds to an unstable equilibrium. In a stable equilibrium, the best response function (of an individual creditor to her peers) intersects the 45^0 line at a slope of less than 1. Let an individual creditor's threshold be θ^{j**} and her peers' threshold be θ^{**}. So

$$\int_{\theta^{**}-\gamma\cdot\Phi^{-1}(\frac{L}{F})}^{+\infty} D(\theta)\cdot f(\theta|P,\theta^{j}=\theta^{j**};\theta^{*})d\theta = F. \tag{A2}$$

We write the LHS of (A2) as the function $U(\theta^{**}, \theta^{j**})$. In solving the equation $U(\theta^{**}, \theta^{j**}) = F$, we have $\frac{\partial\theta^{j**}}{\partial\theta^{**}} = -\frac{\frac{\partial U}{\partial\theta^{**}}}{\frac{\partial U}{\partial\theta^{j**}}}$. Based on the previous result, U is an increasing function at the lower solution, that is, $U(\theta^{**}+\triangle, \theta^{j**}+\triangle) - U(\theta^{**}, \theta^{j**}) > 0$ for a small positive $\triangle$. So $(\frac{\partial U}{\partial\theta^{**}} + \frac{\partial U}{\partial\theta^{j**}})\triangle > 0$, and thus $\frac{\partial U}{\partial\theta^{j**}} > -\frac{\partial U}{\partial\theta^{**}} > 0$. Therefore, $\frac{\partial\theta^{**j}}{\partial\theta^{**}} = -\frac{\frac{\partial U}{\partial\theta^{**}}}{\frac{\partial U}{\partial\theta^{j**}}} < 1$. Similarly, we can prove that $\frac{\partial\theta^{**j}}{\partial\theta^{**}} > 1$ at the higher solution.

We focus on the unique stable equilibrium based on "equilibrium selection in global games with strategic complementarities" in Frankel, Morris and Pauzner (2003) and Vives (2005, 2013). The stable equilibrium in our model also survives the equilibrium selection criterion (refinement) of Pareto efficiency (which is the equilibrium selection criterion in Goldstein and Pauzner (2005) for the upper dominance region).

The case of no solutions means that the creditor-run game does not have a threshold equilibrium. The "bad equilibrium" is a natural outcome. That is, all creditors use the strategy '*Not roll over*' whatever their private signals, considering that they are observing a very low price P. One may think that in this equilibrium creditors would still use a threshold strategy, but their threshold being $\theta^{**}(P) = +\infty$. In this sense, *the role of short selling attacks is to create noise, breaking down threshold equilibria and causing the 'bad' equilibrium.*

The result - none or two threshold equilibria, and one being stable and the other being unstable - is also the case in Angeletos, Hellwig and Pavan (2007). At the stable equilibrium (corresponding to the lowest solution of θ^{**} in equation (A1)), we have $\frac{\partial U(\theta^{**},P;\theta^*)}{\partial \theta^{**}} > 0$. Also considering that $\frac{\partial U(\theta^{**},P;\theta^*)}{\partial P} > 0$, we obtain the relation $\frac{d\theta^{**}}{dP} = -\frac{\frac{\partial U(\theta^{**},P;\theta^*)}{\partial P}}{\frac{\partial U(\theta^{**},P;\theta^*)}{\partial \theta^{**}}} < 0$. Therefore, the equilibrium threshold curve $\theta^{**}(P)$ is downward sloping. In sum, we can rewrite the creditors' equilibrium strategy as

$$(\theta^j, P) \longmapsto \begin{cases} \textit{Not roll over} & P < \widehat{P} \\ \textit{Not roll over} & P \geq \widehat{P} \ \ \& \ \ \theta^j < \theta^{**}(P) \\ \textit{Roll over} & P \geq \widehat{P} \ \ \& \ \ \theta^j \geq \theta^{**}(P) \end{cases},$$

where $\widehat{P}$ is the lowest P for equation (A1) to have solutions, and $\theta^{**}(P)$ is the lowest solution if (A1) admits solutions. Figure A-6 shows the threshold curve $\theta^{**}(P)$.

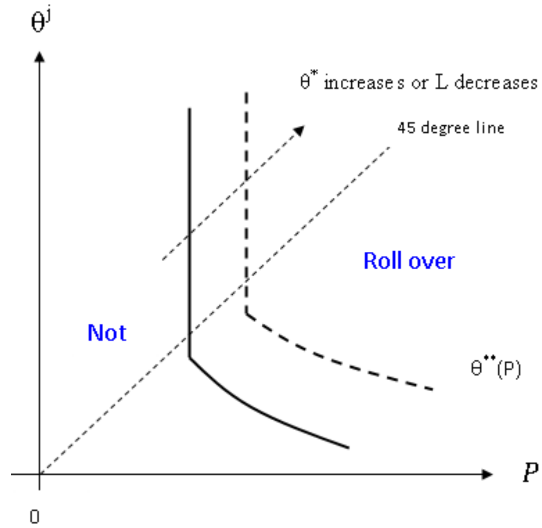

Figure A-6: Creditors' strategy

Finally, we conduct the comparative static analysis. Clearly, $U(\theta^{**}, P; \theta^*)$ is an increasing function in L. That is, when L decreases, $U(\theta^{**}, P; \theta^*)$ decreases. Keeping θ^{**}, P has to increase in order to restore the equation $U(\theta^{**}, P; \theta^*) = F$. That is, the curve $\theta^{**}(P)$ shifts to the right when L decreases. Also, $\widehat{P}$ increases when L decreases under general parameter conditions.

An increase in θ^* not only increases the variance of $f(\theta|P, \theta^j = \theta^{**}; \theta^*)$ but also changes its mean. The latter change is because the weight between the public and private information changes (i.e., the weight of public information goes down). However, in the extreme case $\frac{\gamma}{h} = +\infty$, the second effect disappears and only the first effect exists. Therefore, if $\frac{\gamma}{h}$ is sufficiently high, the first effect dominates the second effect. That is, an increase in θ^* reduces $U(\theta^{**}, P; \theta^*)$. To restore the equation $U(\theta^{**}, P; \theta^*) = F$, P has to increase for a given θ^{**}. That is, the curve $\theta^{**}(P)$ shifts to the right when θ^* increases. Also, $\widehat{P}$ increases when θ^* increases under general parameter conditions; this means that more aggressive short selling causes the 'bad' equilibrium to be more likely.

Considering that when there is no threshold equilibrium the creditors' threshold can be regarded as $\theta^{**}(P) = +\infty$, the comparative static results above can be summarized as $\frac{\partial\theta^{**}(P)}{\partial\theta^*} \geq 0$ and $\frac{\partial\theta^{**}(P)}{\partial L} \leq 0$, that is, given P, the threshold θ^{**} increases in θ^* and decreases in L. Alternatively, we can state that given a private signal θ^j, the threshold of the public signal P increases in θ^* and decreases in L.

Proof of Lemma 2:

We show that $\widehat{\theta}(P)$ decreases in L and increases in θ^*; that is, in the space of Figure A-6, the curve $\widehat{\theta}(P)$ shifts to the right when L decreases or θ^* increases. We have shown that $\theta^{**}(P)$ is decreasing in L and increasing in θ^*. Note that $\widehat{\theta} = \theta^{**} - \gamma \cdot \Phi^{-1}(\frac{L}{F})$ is one-to-one function between θ^{**} and $\widehat{\theta}$. It is straightforward that $\widehat{\theta}(P)$ is increasing in θ^*. Also, because $\theta^{**}(P)$ is decreasing in L and $\gamma \cdot \Phi^{-1}(\frac{L}{F})$ is increasing in L, the two joint forces lead to $\widehat{\theta}(P)$ being decreasing in L.

We compute the bank failure probability, which is given by

$$\Pr(\theta; \theta^*, L) = \begin{cases} \Pr(\theta < \widehat{\theta}(P); \theta^*, L) & \text{if } \theta \leq \widehat{\theta}(\widehat{P}) \\ \Pr(P < \widehat{P}; \theta^*, L) & \text{if } \theta > \widehat{\theta}(\widehat{P}) \end{cases}.$$

Note that $\Pr(\theta < \widehat{\theta}(P); \theta^*, L) = \Pr(P < \widehat{\theta}^{-1}(\theta) | \theta; \theta^*, L)$; intuitively, for a given θ, $\Pr(\theta < \widehat{\theta}(P); \theta^*, L)$ is equal to the probability of P falling below $\widehat{\theta}^{-1}(\theta)$. Therefore, we have that $\Pr(\theta; \theta^*, L) = \begin{cases} \Phi(\frac{\widehat{\theta}^{-1}(\theta) - \theta}{h \cdot s(\theta; \theta^*)}) & \text{if } \theta \leq \widehat{\theta}(\widehat{P}) \\ \Phi(\frac{\widehat{P} - \theta}{h \cdot s(\theta; \theta^*)}) & \text{if } \theta > \widehat{\theta}(\widehat{P}) \end{cases}$, considering that $P \sim N(\theta, (h \cdot s(\theta; \theta^*))^2)$.

Because the threshold curve $\widehat{\theta}(P)$ shifts to the right when L decreases, we have that $\Pr(\theta; \theta^*, L)$ decreases in L.

We check how $\Pr(\theta; \theta^*, L)$ evolves as θ^* changes. Figure A-7 depicts the result.

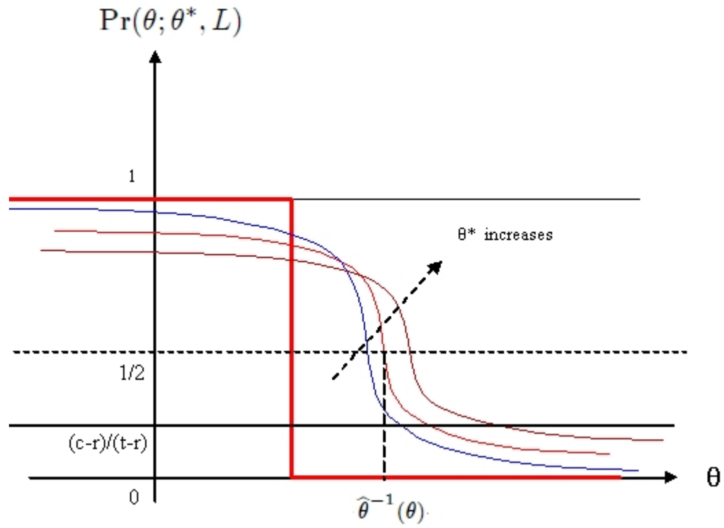

Figure A-7: Bank failure probability

Mathematically, an increase in θ^* has two effects: changing both the threshold $\widehat{\theta}^{-1}(\theta)$ and the standard deviation $h \cdot s(\theta;\theta^*)$. Geometrically, the increase in θ^* not only causes the cdf curve $\Pr(\theta;\theta^*,L)$ to shift to the right (the first effect), but also 'squeezes' the curve to make it flatter (the second effect). Clearly, when θ is high enough such that $\Pr(\theta;\theta^*,L) \leq \frac{1}{2}$, both effects are positive and, hence, $\Pr(\theta;\theta^*,L)$ is certainly increasing in θ^*. Also, if $\frac{F}{K}$ is high, the first effect is dominant. The higher the $\frac{F}{K}$, the wider the region of θ in which $\Pr(\theta;\theta^*,L)$ is increasing in θ^*.

Formally, we have $\frac{\partial\Phi\left(\frac{\widehat{\theta}^{-1}(\theta)-\theta}{h\cdot\Phi(\frac{\theta^*-\theta}{\delta})}\right)}{\partial\theta^*} = \phi\left(\frac{\widehat{\theta}^{-1}(\theta)-\theta}{h\cdot\Phi(\frac{\theta^*-\theta}{\delta})}\right)\left[\frac{\frac{d\widehat{\theta}^{-1}(\theta)}{d\theta^*}}{h\cdot\Phi(\frac{\theta^*-\theta}{\delta})} - \frac{\widehat{\theta}^{-1}(\theta)-\theta}{(h\cdot\Phi(\frac{\theta^*-\theta}{\delta}))^2}\cdot\frac{h}{\delta}\cdot\phi(\frac{\theta^*-\theta}{\delta})\right]$. Clearly, $\frac{d\widehat{\theta}^{-1}(\theta)}{d\theta^*} > 0$ and $\widehat{\theta}^{-1}(\theta)-\theta$ can be positive or negative. If $\theta \geq \widehat{\theta}^{-1}(\theta)$, which corresponds to $\Pr(\theta;\theta^*,L) \leq \frac{1}{2}$, then it is certain that $\frac{\partial\Phi\left(\frac{\widehat{\theta}^{-1}(\theta)-\theta}{h\cdot\Phi(\frac{\theta^*-\theta}{\delta})}\right)}{\partial\theta^*} > 0$. If $\theta < \widehat{\theta}^{-1}(\theta)$, the effects are mixed. However, when θ is close to $\widehat{\theta}^{-1}(\theta)$, the term $\widehat{\theta}^{-1}(\theta)-\theta$ is close to 0. So the total effect is still positive. Also, if $\frac{F}{K}$ is high, the relevant payoffs of creditors become very flat. The upside risk becomes less while the downside risk doesn't change much. Creditors demand a higher 'premium'. A tiny increase in θ^* leads to a big increase in $\widehat{\theta}^{-1}(\theta)$. The first effect dominates the second effect. A similar argument applies when $\Pr(\theta;\theta^*,L) = \Phi(\frac{\widehat{P}-\theta}{h\cdot s(\theta;\theta^*)})$.

Proof of Lemma 3:

We can rewrite equation (5) as

$$\int_{-\infty}^{+\infty} \Pr(\theta;\theta^*,L)\cdot dG(\theta|\theta^{i*}) = \frac{c-r}{t-r}. \tag{A3}$$

Note that $\Pr(\theta;\theta^*,L)$ is decreasing in θ and is between 0 and 1. Because $\frac{c-r}{t-r}<1$, there exists a unique solution of θ^{i*} in (A3).

From the proof of Lemma 2, we have the following result: There exists a value Y such that for any $y\in[0,Y]$, θ is increasing with θ^* in solving the implicit function $\Pr(\theta;\theta^*,L)=y$. That is, in Figure A-7, for the bottom part of curves such that $\Pr(\theta;\theta^*,L)\leq Y$, the curves shift to the right as θ^* increases.

Based on the proof of Lemma 2, Y is greater than $\frac{1}{2}$ and increases in $\frac{F}{K}$. Therefore, if $\frac{c-r}{t-r}$ is small enough (e.g. $\frac{c-r}{t-r}\leq\frac{1}{2}$) or $\frac{F}{K}$ is high enough, θ^{i*} is increasing with θ^* in solving (A3).

Proof of Lemma 4:

First, we have the result that the influence of short selling on creditors' decisions is limited. In fact, even at the extreme of $\theta^*=+\infty$ in which case every speculator participates in the short-selling attack and $s=1$, P still has a limited volatility, that is, $P\sim N(\theta,h^2)$.

Considering that short selling can influence the decisions of the creditors to a certain degree, which is not unlimited, and all speculators realize this, when a speculator receives a very strong signal θ^i, she is not going to short sell whatever her beliefs about other speculators' actions; when her signal θ^i is very weak, she is going to short sell whatever her beliefs about other speculators' actions. Formally, there exists a lower boundary $\underline{\theta}$ and an upper boundary $\overline{\theta}$ such that a speculator definitely short sells if her signal $\theta^i<\underline{\theta}$ and definitely does not short sell if $\theta^i>\overline{\theta}$, whatever her belief about other speculators' actions and whatever her belief about creditors' beliefs about speculators' actions. That is, a speculator's threshold θ^* can never be outside the interval $[\underline{\theta},\overline{\theta}]$.

Also, by Lemma 3, $\theta^{i*}(\theta^*)$ is increasing in θ^* within some interval $\theta^*\in(\theta^L,\theta^H)$.

Based on the above results, Figure A-8 illustrates the proof. In the figure, the unique intersection of the curve $\theta^{i*}(\theta^*)$ and the 45^0 line represents the equilibrium. Consider that $\theta^{i*}(\theta^*)$ is increasing in θ^* for $\theta^*\in(\theta^L,\theta^H)$, and also that θ^{i*} is bounded within $\theta^{i*}\in[\underline{\theta},\overline{\theta}]$. Further, we can show that $[\underline{\theta},\overline{\theta}]\subset(\theta^L,\theta^H)$ if δ is sufficiently high. So we have $\theta^{i*}(\theta^*=\underline{\theta})>\theta^{i*}(\theta^*=\theta^L+)\geq\underline{\theta}$, which means that the point $(\underline{\theta},\ \theta^{i*}(\underline{\theta}))$ is above the 45^0 line. Similarly, we have $\theta^{i*}(\theta^*=\overline{\theta})<\overline{\theta}$, meaning that the point $(\overline{\theta},\ \theta^{i*}(\overline{\theta}))$ is below the 45^0 line. Therefore, there exist intersections between the curve $\theta^{i*}(\theta)$ and the 45^0 line. As for the uniqueness of intersections, we have to rely on simulations as the functions are implicit and we do not have closed-form solutions. The numerical example in Appendix A illustrates one case under a set of parameter values.

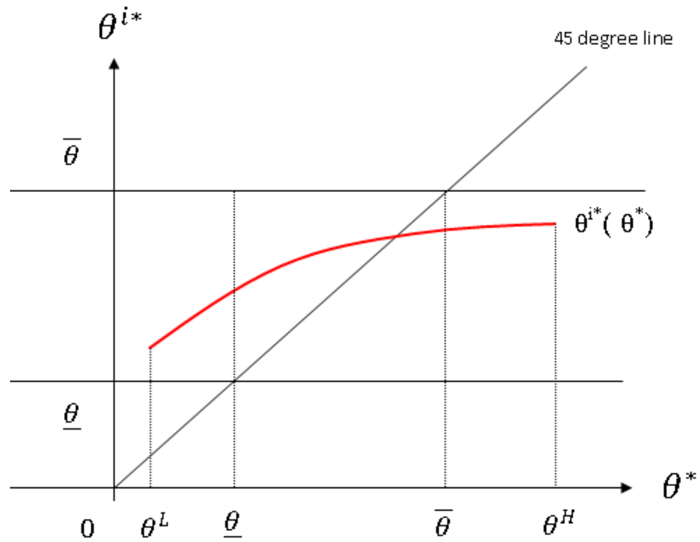

Figure A-8: The equilibrium among speculators

Finally, we show the comparative static result. In solving equation (A3), it is easy to obtain the comparative static result $\frac{\partial \theta^{i*}}{\partial t} > 0$. That is, the curve $\theta^{i*}(\theta^*)$ shifts up when t increases. So, in Figure A-8 the intersection between the curve $\theta^{i*}(\theta^*)$ and the 45^0 line increases in t. That is, the equilibrium threshold θ^* is increasing in t.

Proof of Proposition 3:

In Lemma 1, we prove that $\frac{\partial \theta^{**}(P)}{\partial \theta^*} > 0$. Now we prove that $\frac{\partial \theta^*}{\partial \theta^{**}(P)} > 0$.

When the threshold $\theta^{**}(P)$ increases, so does $\widehat{\theta}(P)$; then the bank failure probability $\Pr(\theta; \theta^*, L)$ increases for each given θ. By equation (A3), therefore, the curve $\theta^{i*}(\theta^*)$ shifts up as $\theta^{**}(P)$ increases. So, in Figure A-8 the intersection between the curve $\theta^{i*}(\theta^*)$ and the 45^0 line increases in $\theta^{**}(P)$. That is, the equilibrium threshold θ^* is increasing in $\theta^{**}(P)$.